Part I | Book 3

FRM® EXAM PREP

SCHWESER 2015

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Financial Markets and Products

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Part I FRM® Exam

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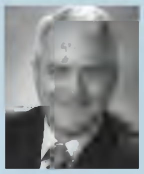

May Exam Instructor
Dr. John Broussard
CFA, FRM

November Exam Instructor
Dr. Greg Filbeck
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FRM Part I Book 3: Financial Markets and Products

FRM 2015 PART I BOOK 3: FINANCIAL MARKETS AND PRODUCTS

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Reading Assignments and Learning Objectives

The following material is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by the Global Association of Risk Professionals.

Reading Assignments

The Institute for Financial Markets, *Futures and Options* (Washington, DC: The Institute for Financial Markets, 2011).

John Hull, *Options, Futures, and Other Derivatives, 9th Edition* (New York: Pearson Prentice Hall, 2014).

Robert McDonald, *Derivatives Markets, 3rd Edition* (Boston: Addison-Wesley, 2013).

Learning Objectives

30. Introduction: Futures and Options Markets

After completing this reading, you should be able to:
1. Evaluate how the use of futures contracts can mitigate common risks in the commodities business. (page 14)
2. Describe the key features and terms of a futures contract. (page 16)
3. Differentiate between equity securities and futures contracts. (page 17)
4. Define and interpret volume and open interest. (page 18)
5. Explain the requisites for a successful futures market. (page 18)

31. Futures Industry Institutions and Professionals

After completing this reading, you should be able to:
1. Describe the features of a modern futures exchange and identify typical contract terms and trading rules. (page 25)
2. Explain the organization and administration of an exchange and clearinghouse. (page 26)
3. Describe exchange membership, the different types of exchange members, and the exchange rules for member trading. (page 27)
4. Explain original and variation margin, daily settlement, the guaranty deposit, and the clearing process. (page 27)
5. Summarize the steps that are taken when a clearinghouse member is unable to meet its financial obligations on its open contracts. (page 29)
6. Describe the mechanics of futures delivery and the roles of the clearinghouse, buyers, and sellers in this process. (page 30)
7. Explain the role of futures commission merchants, introducing brokers, account executives, commodity trading advisors, commodity pool operators, and customers. (page 32)

32. Hedging with Futures and Options

After completing this reading, you should be able to:
1. Define the terms "long the basis" and "short the basis." (page 42)
2. Explain exchange for physical (EFP) transactions and their role in the energy and financial futures markets. (page 44)
3. Outline and calculate the payoffs on the various scenarios for hedging with options on futures. (page 45)

33. Introduction (Options, Futures, and Other Derivatives)

After completing this reading, you should be able to:
1. Differentiate between an open outcry system and electronic trading. (page 52)
2. Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages (page 52)
3. Differentiate between options, forwards, and futures contracts. (page 53)
4. Identify and calculate option and forward contract payoffs. (page 53)
5. Calculate and compare the payoffs from hedging strategies involving forward contracts and options. (page 57)
6. Calculate and compare the payoffs from speculative strategies involving futures and options. (page 59)
7. Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary. (page 62)
8. Describe some of the risks that can arise from the use of derivatives. (page 62)

34. Mechanics of Futures Markets

After completing this reading, you should be able to:

1. Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery and limits. (page 68)
2. Explain the convergence of futures and spot prices. (page 70)
3. Describe the rationale for margin requirements and explain how they work. (page 70)
4. Describe the role of a clearinghouse in futures and over-the-counter market transactions. (page 71)
5. Describe the role of collateralization in the over-the-counter market and compare it to the margining system. (page 72)
6. Identify the differences between a normal and inverted futures market. (page 73)
7. Describe the mechanics of the delivery process and contrast it with cash settlement. (page 73)
8. Evaluate the impact of different trading order types. (page 74)
9. Compare and contrast forward and futures contracts. (page 68)

35. Hedging Strategies Using Futures

After completing this reading, you should be able to:

1. Define and differentiate between short and long hedges and identify their appropriate uses. (page 80)
2. Describe the arguments for and against hedging and the potential impact of hedging on firm profitability. (page 80)
3. Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures. (page 81)
4. Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness. (page 81)
5. Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the "tailing the hedge" adjustment. (page 84)
6. Explain how to use stock index futures contracts to change a stock portfolio's beta. (page 85)
7. Explain the term "rolling the hedge forward" and describe some of the risks that arise from this strategy. (page 86)

36. Interest Rates

After completing this reading, you should be able to:

1. Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the "risk-free" rate. (page 92)
2. Calculate the value of an investment using different compounding frequencies. (page 93)
3. Convert interest rates based on different compounding frequencies. (page 93)
4. Calculate the theoretical price of a bond using spot rates. (page 94)
5. Calculate forward interest rates from a set of spot rates. (page 98)
6. Calculate the value of the cash flows from a forward rate agreement (FRA). (page 99)
7. Calculate the duration, modified duration and dollar duration of a bond. (page 100)
8. Evaluate the limitations of duration and explain how convexity addresses some of them. (page 101)

9. Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates. (page 102)
10. Compare and contrast the major theories of the term structure of interest rates. (page 103)

37. Determination of Forward and Futures Prices

After completing this reading, you should be able to:

1. Differentiate between investment and consumption assets. (page 108)
2. Define short-selling and calculate the net profit of a short sale of a dividend-paying stock. (page 108)
3. Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices. (page 109)
4. Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices. (page 109)
5. Explain the relationship between forward and futures prices. (page 113)
6. Calculate a forward foreign exchange rate using the interest rate parity relationship. (page 112)
7. Define income, storage costs, and convenience yield. (page 114)
8. Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields. (page 114)
9. Define and calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows. (page 109)
10. Describe the various delivery options available in the futures markets and how they can influence futures prices. (page 115)
11. Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk. (page 115)
12. Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model. (page 116)

38. Interest Rate Futures

After completing this reading, you should be able to:

1. Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation. (page 121)
2. Calculate the conversion of a discount rate to a price for a US Treasury bill. (page 123)
3. Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond. (page 122)
4. Explain and calculate a US Treasury bond futures contract conversion factor. (page 124)
5. Calculate the cost of delivering a bond into a Treasury bond futures contract. (page 124)
6. Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision. (page 124)
7. Calculate the theoretical futures price for a Treasury bond futures contract. (page 125)
8. Calculate the final contract price on a Eurodollar futures contract. (page 127)
9. Describe and compute the Eurodollar futures contract convexity adjustment. (page 127)
10. Explain how Eurodollar futures can be used to extend the LIBOR zero curve. (page 128)

11. Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures. (page 128)
12. Explain the limitations of using a duration-based hedging strategy. (page 129)

39. Swaps

After completing this reading, you should be able to:

1. Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows. (page 135)
2. Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows. (page 136)
3. Explain the role of financial intermediaries in the swaps market. (page 136)
4. Describe the role of the confirmation in a swap transaction. (page 136)
5. Describe the comparative advantage argument for the existence of interest rate swaps and explain some of the criticisms of this argument. (page 137)
6. Explain how the discount rates in a plain vanilla interest rate swap are computed. (page 138)
7. Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions. (page 138)
8. Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs). (page 140)
9. Explain the mechanics of a currency swap and compute its cash flows. (page 142)
10. Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows. (page 144)
11. Calculate the value of a currency swap based on two simultaneous bond positions. (page 142)
12. Calculate the value of a currency swap based on a sequence of FRAs. (page 143)
13. Describe the credit risk exposure in a swap position. (page 145)
14. Identify and describe other types of swaps, including commodity, volatility and exotic swaps. (page 145)

40. Mechanics of Options Markets

After completing this reading, you should be able to:

1. Describe the types, position variations, and typical underlying assets of options. (page 152)
2. Explain the specification of exchange-traded stock option contracts, including that of nonstandard products. (page 158)
3. Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options. (page 160)

41. Properties of Stock Options

After completing this reading, you should be able to:

1. Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options. (page 167)
2. Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks. (page 169)
3. Explain put-call parity and apply it to the valuation of European and American stock options. (page 170)
4. Explain the early exercise features of American call and put options. (page 172)

42. Trading Strategies Involving Options

After completing this reading, you should be able to:

1. Explain the motivation to initiate a covered call or a protective put strategy. (page 179)
2. Describe the use and calculate the payoffs of various spread strategies. (page 180)
3. Describe the use and explain the payoff functions of combination strategies. (page 185)

43. Exotic Options

After completing this reading, you should be able to:

1. Define and contrast exotic derivatives and plain vanilla derivatives. (page 195)
2. Describe some of the factors that drive the development of exotic products. (page 195)
3. Explain how any derivative can be converted into a zero-cost product. (page 196)
4. Describe how standard American options can be transformed into nonstandard American options. (page 196)
5. Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options. (page 197)
6. Describe and contrast volatility and variance swaps. (page 200)
7. Explain the basic premise of static option replication and how it can be applied to hedging exotic options. (page 201)

44. Commodity Forwards and Futures

After completing this reading, you should be able to:

1. Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield. (page 206)
2. Explain the basic equilibrium formula for pricing commodity forwards. (page 206)
3. Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit. (page 208)
4. Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures. (page 211)
5. Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds. (page 213)
6. Compute the forward price of a commodity with storage costs. (page 213)
7. Compare the lease rate with the convenience yield. (page 215)
8. Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices. (page 215)
9. Compute a commodity spread. (page 218)
10. Explain how basis risk can occur when hedging commodity price exposure. (page 218)
11. Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management. (page 219)
12. Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives. (page 220)
13. Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price. (page 206)

45. Foreign Exchange Risk

After completing this reading, you should be able to:
1. Calculate a financial institution's overall foreign exchange exposure. (page 227)
2. Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk. (page 227)
3. Calculate a financial institution's potential dollar gain or loss exposure to a particular currency. (page 227
4. Identify and describe the different types of foreign exchange trading activities. (page 228)
5. Identify the sources of foreign exchange trading gains and losses. (page 229)
6. Calculate the potential gain or loss from a foreign currency denominated investment. (page 229)
7. Explain balance-sheet hedging with forwards. (page 231)
8. Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates. (page 234)
9. Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk. (page 235)
10. Describe the relationship between nominal and real interest rates. (page 235)

46. Corporate Bonds

After completing this reading, you should be able to:
1. Describe a bond indenture and explain the role of the corporate trustee in a bond indenture. (page 241)
2. Explain a bond's maturity date and how it impacts bond retirements. (page 241)
3. Describe the main types of interest payment classifications. (page 242)
4. Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk. (page 242)
5. Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds. (page 243)
6. Describe the mechanisms by which corporate bonds can be retired before maturity. (page 245)
7. Differentiate between credit default risk and credit spread risk. (page 246)
8. Describe event risk and explain what may cause it in corporate bonds. (page 247)
9. Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds. (page 247)
10. Define and differentiate between an issuer default rate and a dollar default rate. (page 248)
11. Define recovery rates and describe the relationship between recovery rates and seniority. (page 249)

47. Mortgages and Mortgage-Backed Securities

After completing this reading, you should be able to:
1. Describe the various types of residential mortgage products. (page 254)
2. Calculate a fixed rate mortgage payment, and its principal and interest components. (page 257)
3. Describe the mortgage prepayment option and the factors that influence prepayments. (page 260)

4. Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs. (page 261)
5. Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool. (page 261)
6. Describe a dollar roll transaction and how to value a dollar roll. (page 266)
7. Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments. (page 269)
8. Describe the steps in valuing an MBS using Monte Carlo Simulation. (page 271)
9. Define Option Adjusted Spread (OAS), and explain its challenges and its uses. (page 274)

48. The Rating Agencies

After completing this reading, you should be able to:

1. Describe the role of rating agencies in the financial markets. (page 280)
2. Explain market and regulatory forces that have played a role in the growth of the rating agencies. (page 280)
3. Describe a rating scale, define credit outlooks, and explain the difference between solicited and unsolicited ratings. (page 281)
4. Describe Standard and Poor's and Moody's rating scales and distinguish between investment and noninvestment grade ratings. (page 281)
5. Describe the difference between an issuer-pay and a subscriber-pay model and describe concerns regarding the issuer-pay model. (page 282)
6. Describe and contrast the process for rating corporate and sovereign debt and describe how the distributions of these ratings may differ. (page 283)
7. Describe the relationship between the rating agencies and regulators and identify key regulations that impact the rating agencies and the use of ratings in the market. (page 284)
8. Describe some of the trends and issues emerging from the recent credit crisis relevant to the rating agencies and the use of ratings in the market. (page 285)

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Introduction: Futures and Options Markets

Topic 30

Exam Focus

This topic is an introduction to futures and options markets. In addition to providing futures contracts for agricultural commodities, futures markets have expanded to include financial instruments and industrial raw materials such as copper, oil, and natural gas. Modern futures markets make it possible to hedge any existing economic risk. For the exam, understand the features and terms of futures contracts and how they differ from equity securities. In addition, know how modern futures markets function and what is needed for a successful futures market.

Risks in the Commodities Business

LO 30.1: Evaluate how the use of futures contracts can mitigate common risks in the commodities business.

The futures market was established to satisfy the need to manage price risk in an agriculturally driven economy. The "to-arrive" contract was the first attempt to deal with the price change risk associated with commodities. The terms of the sale were agreed to in advance, and the deal was only finalized when the goods arrived.

Modern futures markets allow producers, distributors, dealers, and investors to manage the uncertainty of prices over time and participate in efficient price discovery. In addition to supply/demand price risks, other commodities business risks can also be addressed through the use of **futures contracts**, including storage problems, variations in quality, lack of price transparency, lack of standard payment terms, resale problems, counterparty risk, and lack of standardized contracts.

Storage

The buyer of a commodity has to trust that the seller can deliver the promised goods. The buyer must be confident that the goods are available and the seller must be confident that the buyer can pay for them. In addition, the buyer must also have confidence that issues related to the handling of commodities, such as spoilage and fraud, are minimized. Futures exchanges have played a major role in the development of laws and regulations governing the handling of commodities and have developed systems for the licensing and inspection of warehouses used to store commodities.

Quality Risk

The Chicago Board of Trade (CBOT) introduced grading, weighing, and measurement standards in the nineteenth century, which evolved into the standardization of deliverable grades that is at the heart of futures trading today. This standardization allows for a more efficient and trusted system of trading. As with most futures exchanges, procedures and systems that started out for agricultural products have expanded to other commodities and financial instruments.

Price Discovery and Standardized Payment

Before the use of futures contracts and exchanges, price discovery was not very efficient and sometimes not even possible. With futures exchanges, price dissemination is accomplished by requiring that (1) buyers and sellers communicate competitive bids and offers from a single physical or electronic location and (2) completed transactions are immediately communicated to a wide audience. In addition to effective price discovery, current futures exchanges are able to standardize payment methods. All payments must be made with cash, and all trades must be cleared through members of the exchange who meet exchange-approved standards.

Offsetting Transactions

Most hedgers and speculators have no interest in taking delivery of an actual commodity when they buy or sell futures contracts. Speculators are attempting to take advantage of price discrepancies, and hedgers are looking to transfer risk. These traders often have little interest in the grade or quality of the underlying commodity or financial instrument. The ability to buy or sell futures contracts quickly and easily is crucial for an efficient and effective market. Participants must be able to sell (buy) existing long (short) positions, which closes out (i.e., offsets) a position, even if the trade is not with the original counterparty. Futures clearinghouses allow for offsetting transactions by guaranteeing that the holder of the long position (the buyer) is able to sell to any willing buyer in the futures markets. The innovation of the clearinghouse, which is discussed in the next topic, allows for a liquid, low-cost futures market.

Counterparty Risk

Another major risk associated with any market, but especially for the futures market, is counterparty risk. Both sides of a trade have to be confident that the other party will perform as expected. This is another risk addressed by the clearinghouse. The clearinghouse guarantees the financial integrity of every futures contract, acting as a financial intermediary on each cleared trade.

Trading Risk

Trading risks, like fraud, can be minimized by having standardized contracts under published exchange rules that describe methods of operations and permitted trading procedures. Futures exchanges provide efficient functioning markets and lessen these trading risks.

Futures Contract Features and Terms

LO 30.2: Describe the key features and terms of a futures contract.

A futures contract is an agreement between two parties to buy or sell a preset quantity and grade of a specified item, such as a commodity, security, currency, or index, at an agreed-upon price on or before a given date in the future. Futures have several key features including:

- The buyer is long the futures contract and receives delivery.
- The seller is short the futures contract and makes delivery.
- Offsetting (opposite) transactions can extinguish (close) futures contract positions.
- Settlement can be by physical delivery or in cash, depending on contract specifications.
- Futures on the same or similar commodities can be traded on more than one exchange.

The standardized terms of futures contracts are set by the exchanges on which the contracts trade. Standardized terms include the following:

- *Underlying instrument* (also called the spot instrument). The physical commodity, security, index, or currency underlying the contract, often called physicals, actuals, or cash.
- *Contract size.* Determines how much of the underlying the contract represents.
- *Settlement mechanism* (by cash or physical delivery). If by physical delivery, delivery location and other requirements are specified.
- *Delivery date* (also called maturity date). The contractual date when the buyer is obligated to pay the seller and the seller is obligated to deliver to the buyer, or the date when cash settlement takes place.
- *Grade or quality.* Occasionally, delivery of a grade higher or lower than the specified grade is permitted at a premium or discount to the futures contract grade.

The exchange establishes the months that contracts may be traded and delivered. Some months are more active than others for trading certain commodities.

If a contract is held to maturity, and it is not a cash settlement contract, then the long trader must accept the commodity according to the grade and location designated by the contract, whether he wants it or not.

When physical commodities have been inspected and approved for delivery, they are known as "certified stock." Inventory level information is available from the exchange where the futures contracts are traded. Unexpected decreases or increases in the certified stock of a particular commodity can produce an immediate market response.

Equity Securities vs. Futures Contracts

LO 30.3: Differentiate between equity securities and futures contracts.

Futures have many unique features and differ from forward contracts and equity securities in several different ways. Distinct differences between futures contracts and equities (i.e., stocks) are as follows:

- Futures markets are designed more for risk shifting and price discovery than capital formation. The equity market's main purpose is to assist in capital formation.
- Shorting is easier in the futures market than the equity market. There is a short for every long in the futures market; therefore, it is just as easy to take a short position as it is to take a long position. Shorting stock requires borrowing shares and other complications, such as payment of dividends.
- Futures contracts have a finite time frame, whereas equities do not.
- The margin for stocks is a down payment, and interest is charged on the remaining loan balance. The margin required for a futures position is earnest money, a promise to fulfill the contract's obligation.
- Futures markets generally have limits on price and positions, which establishes a range of trading prices and maximum positions allowed for a given futures instrument. Individual stocks generally do not have limits on price movement or position size.
- There are a finite number of shares issued on an individual stock at any point in time. There is no limit on the number of futures contracts that may exist at any point in time for a given commodity.
- In most instances, owners of equities can request a stock certificate showing ownership. There is no certificate of ownership for futures contracts. A record of ownership is held with the brokerage house through which the trade was made, and a trade confirmation is usually sent to the client.
- There are significant differences between the non-electronic trading on the futures and the equity markets. The futures markets have an open-outcry system where members compete on the floor of the exchange for the best price. Most stock markets have a specialist that is responsible for maintaining an orderly market on a particular stock. The differences between the two systems can have an impact on transaction prices. Note that the use of electronic trading systems has increased for both futures and stocks.
- The Securities and Exchange Commission (SEC), stock exchanges, state regulatory agencies, and the Financial Industry Regulatory Authority (FINRA) regulate equity securities transactions. Transactions in U.S. futures and options on futures are regulated by the Commodity Futures Trading Commission (CFTC), futures exchanges, and the National Futures Association (NFA).

Unlike futures contracts, forward contracts are not standardized or exchange traded. Forwards are tailored contracts negotiated between private parties (e.g., between an agricultural producer and a grain elevator), between a bank and one of its clients, or between two financial institutions. Forwards are useful when actual delivery is preferred because the contracts can be customized to the needs of the two parties. However, this customization comes with a higher cost and less liquidity than futures contracts. Because forwards are not traded on an exchange, they cannot be liquidated with an offsetting transaction like futures. They are also not guaranteed by a centralized clearinghouse, which introduces counterparty risk into the agreement.

Volume and Open Interest

LO 30.4: Define and interpret volume and open interest.

Every trading day, and even intraday, statistics regarding daily trading volume and open interest are published by the clearinghouse of each exchange from trade data submitted by members. Analysts, traders, and investors attach significant importance to this data.

Volume is the total amount of purchases *or* sales during a trading session (not purchases and sales together). There is a buyer and seller for each contract traded; as a result, total purchases must equal total sales. Trades that cannot be matched with offsetting trades are called an out-trade discrepancy. Out-trades are resolved by the exchange or clearinghouse and not included in the total volume data. If Client A buys one contract of January wheat and Client B sells one contract of January wheat, the volume of trading between them is recognized as just one contract, not two.

The total number of futures contracts that remain open at the end of a trading session is known as **open interest.** Open interest includes those contracts not yet liquidated by either an offsetting futures market transaction or delivery. If Client A (the long position) buys one contract of January wheat from Client B (the short position), and neither client started with a position in January wheat, one futures contract will be created and open interest will increase by one.

If Client A, the holder of an existing long position, sells his position, open interest will decrease by one if it is sold to Client B, who is currently holding an existing short position. In this case, Client B would be offsetting or covering his existing short position. If Client A's long position is instead sold to a new buyer, Client C, there will be no change to open interest. In this case, Client C simply replaces Client A as the holder of the long position.

Futures Market Essentials

LO 30.5: Explain the requisites for a successful futures market.

Futures markets for some commodities are more successful than others. What makes some commodity futures more successful is the existence of real economic risk. If producers and users of commodities need to manage economic risk, there is a better chance of a successful futures market. If there is little volatility in the price of a commodity, there is little incentive to trade or manage risk.

Volatility can be dampened by natural market forces but also by government action. Rigid government controls, like during World War II, take away the need for a futures markets. When governments set price controls, there is no need for hedging, and as a result, no need for a futures market.

In addition to the existence of risk, participants must be willing to use the futures markets to manage risk. For market participants, the following items are necessary:

- *An available underlying cash market.* Futures contracts must be based on real supply and demand economics. For example, economic activity such as the stock market, the stockyards, grain elevators, or the mortgage market must underlie an effective futures market.
- *Transparency.* Information regarding the underlying economic activity, the supply and demand, must also be widely available. If the underlying cash market is opaque, the corresponding futures market has less chance of success.
- *Standardization.* The grade or type of commodity must meet the quality standards called for in the contract.
- *Efficient delivery infrastructure.* Exchanges have made delivery easier with cash-settled contracts.
- *No other liquid contracts.* No other liquid futures contracts can be available to hedge the same or similar risks.

Uses of Futures and Options

Participants use futures and options for two broad reasons: hedging and speculating. Hedging is used for risk management. The hedger has a risk associated with the underlying commodity or financial instrument. The use of futures helps mitigate those risks. Speculating does not mitigate risk but is risk-taking. Profit is the motive of the speculator since he has no risk before entering into the futures transactions. Both of these types of trading activity can take on different forms, including hedging, carrying of commodity inventory, arbitrage, position taking, price discovery, and speculation.

The principal economic activity of **hedgers** is to produce, distribute, process, store, or invest in an underlying commodity or financial instrument. They have a need to take a position in the futures market that is opposite an existing position in the cash market. A producer of wheat would sell a futures contract if they wish to be neutral or indifferent to changes in the cash market price of a commodity. Like other forms of insurance, hedging can lower business costs and allow risk to be taken in other areas. Hedging reduces or eliminates price risk by shifting it to others with opposite risk profiles or to speculators who wish to take on risk for profit.

Those who carry an inventory of a commodity can use futures to cover the carry costs. Once harvested, a farm crop must be carried until consumption. Futures prices often include all or part of the cost of storage, insurance, and interest payments to carry the commodity until expiration. A short position in the commodity can cover some of these costs. Similarly, a commodity user can take a long position to carry a commodity until needed. The cost to the user to carry the long futures position will be the same or possibly less than holding the physical inventory.

Arbitrage ensures that futures and cash markets stay in balance. Buying in the cheaper market and selling in the overpriced market will bring markets back into alignment and provide a riskless profit for an arbitrageur. That is, arbitrage helps keep markets closer to their fair market value.

Position taking can be a form of speculating or hedging. Taking a futures position without an offsetting cash position would be considered speculation. Taking a futures position in

anticipation of a future risk is considered hedging. For example, a company planning on borrowing funds in the future may wish to hedge interest rate risk by locking in borrowing costs.

A competitive futures market helps establish a single known price (i.e., participants can discover what a commodity is worth). The openness of the futures market and the speed at which prices are disseminated puts all participants on a level playing field. In addition, assessing risk is critical for hedging and speculating, and assessing risk cannot happen without effective price discovery. In the futures and options markets, participants are able to plan, hedge risk, and take on new risk as appropriate with accurate prices and statistics.

Speculators take on risk in an attempt to earn a positive return. The risk absorbed by speculators comes from other speculators or from hedgers. Hedgers have existing risk to transfer, and speculators simply take the other side of the trade. Participants acting in their own self-interest make the futures market function, providing liquidity and price discovery with minimal price disturbance. Long-term speculators help smooth price fluctuations by injecting capital into the marketplace.

Key Concepts

LO 30.1

Risks in the commodities business that can be addressed through the use of futures contracts include supply/demand price risks, storage problems, variations in quality, lack of standard payment terms, lack of price transparency, resale problems, counterparty risk, and lack of standardized contracts.

LO 30.2

Key features of futures contracts include:

- The buyer is long the futures contract and receives delivery.
- The seller is short the futures contract and makes delivery.
- Offsetting (opposite) transactions can extinguish (close) futures contract positions.
- Settlement can be by physical delivery or in cash, depending on exchange specifications.
- Futures on the same or similar commodities can be traded on more than one exchange.

Standardized terms of futures contracts include:

- Underlying instrument.
- Contract size.
- Settlement mechanism.
- Delivery date.
- Grade or quality.

LO 30.3

There are several differences between futures and equities including:

- Futures markets are designed more for risk shifting and price discovery than capital formation. The equity market's main purpose is to assist in capital formation.
- Shorting is easier in the futures market than the equity market.
- Futures markets generally have limits on price and positions, establishing a range of trading prices and maximum positions allowed for a given futures instrument. Individual stocks generally do not have limits on price movement or position size.
- There are a finite number of shares issued on an individual stock at any point in time. There is no limit on the number of futures contracts that may exist at any point in time for a given commodity.

LO 30.4

Volume is the total purchases or sales during a trading session. The total number of futures contracts that remain open at the end of a trading session is known as open interest. Open interest includes contracts not yet liquidated either by an offsetting futures market transaction or delivery.

LO 30.5

What makes some commodity futures markets more successful than others is the existence of real economic risk. If producers and users of commodities need to manage economic risk, there is a better chance of a successful futures market. If there is little volatility in the price of a commodity, there is little incentive to trade or manage risk. In addition to a real economic need, willing participants are also a requisite for a successful futures market.

Concept Checkers

1. Which of the following types of risk is least likely to be a source of risk in the commodities markets?
   A. Earnings risk.
   B. Price risk.
   C. Counterparty risk.
   D. Quality risk.

2. BullsEye, Inc., is planning to issue new debt securities next year. The company is concerned that interest rates may rise, which will increase its borrowing costs. Which of the following types of traders best describe the role BullsEye should consider in the futures markets?
   A. Arbitrageur.
   B. Hedger.
   C. Speculator.
   D. Market maker.

3. Client A buys one July wheat contract from Client B. On the same day, Client B buys the same wheat contract from Client C. Assuming open interest started out at zero, and no deliveries were made, what is the addition to open interest for July wheat contracts?
   A. 0.
   B. 1.
   C. 2.
   D. 3.

4. Farm A recently produced a record amount of spring wheat and is concerned about a potential severe decline in prices. Historically, spring wheat prices have been very volatile, subject to large price swings. The market has gone from drought to record crops in just a five-year period. The large swings in prices have influenced the local government to recently enact price controls for the indefinite future. Farm B, the farm next to Farm A, has suffered some large losses throughout the last five years due to the recent price volatility. Farm A is considering buying Farm B, hoping to capitalize on that farm's recent troubles. Which of the following trades represent an appropriate futures hedge for Farm A?
   A. Long spring wheat contracts.
   B. No hedge.
   C. Short spring wheat contracts.
   D. Long winter wheat contracts.

5. Which of the following statements is least likely a difference between the futures and equity markets?
   A. The equity markets are mostly for capital formation.
   B. The futures market has a finite time frame.
   C. Shorting is easier in the equity markets.
   D. There is no limit for the number of futures contracts outstanding.

Concept Checker Answers

1. **A** Modern futures markets allow producers, distributors, dealers, and investors to manage the uncertainty of prices over time and participate in efficient price discovery. In addition to supply/demand price risk, risk can also come from storage problems, quality risk, lack of standard payment terms, lack of price transparency, resale problems, counterparty risk, and lack of standardized contracts.

2. **B** BullsEye should consider hedging interest rate risk. When BullsEye issues debt, it will have an existing risk and will desire an offsetting position to reduce or eliminate that risk. A speculator would not have an existing position to hedge; they would be looking to initiate a risky position not eliminate one. An arbitrageur would wish to take advantage of a market mispricing with a riskless transaction.

3. **B** Open interest includes contracts not yet liquidated either by an offsetting futures market transaction or delivery. When Client A buys one contract from Client B, the current open interest is one, assuming neither party started with a position in that contract. When Client B buys the same contract from Client C, Client B closes out their short transaction with an offsetting long transaction. The open interest would not change because Client C would simply take the place of Client B. Client A would still be long one contract, and now Client C would be short one contract. Thus, open interest would be one.

4. **B** One of the requisites for a successful futures market is that the producers and users of commodities need to manage economic risk. If there is little volatility in the price of a commodity, there is little incentive to trade or manage risk. If the government enacts price controls, there would be little need for participants to hedge. Given the price controls, Farm A would no longer need to hedge its long wheat position because the future price of wheat would be known.

5. **C** Shorting is easier in the futures market than the equity market. There is a short for every long in the futures market; therefore, it is just as easy to take a short position as it is to take a long position. Shorting stocks requires borrowing shares and other complications, such as payment of dividends.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Futures Industry Institutions and Professionals

Topic 31

Exam Focus

This topic presents an overview of the institutions and professionals associated with futures exchanges and clearinghouses. Exchange professionals such as futures commission merchants, introducing brokers, account executives, commodity trading advisors, commodity pool operators, and customers all play critical roles in the futures market. For the exam, know how exchange operations and mechanics of futures delivery work, and understand the different roles played by the futures market members, clearinghouse, and customers. Also, be familiar with the clearinghouse's role as the counterparty for every trade.

Futures Contract Terms and Trading Rules

LO 31.1: Describe the features of a modern futures exchange and identify typical contract terms and trading rules.

In the late 1990s, futures exchanges evolved from a not-for-profit member association model to a for-profit model. For-profit futures exchanges generally issue two classes of stock: **Class A shares** represent equity ownership in the exchange and voting rights, and **Class B shares** represent trading rights. Trading rights are often leased or used directly by owners of Class B shares.

The exchanges do not own the underlying instruments, nor do they trade or take positions in futures or options contracts. Their role is to provide electronic or physical trading facilities, rules to conduct trading, and operational mechanisms. Because the exchanges themselves do not take positions, trading is conducted in the secondary market. Similar to stock exchange members, futures exchange members trade amongst themselves for their own accounts or for customer accounts. Futures exchanges differ from stock exchanges in that they design (i.e., author) the contracts that trade on the exchange, writing all terms and conditions of standardized contracts.

Common contract terms and trading rules include:

- Contact/delivery months available for trading.
- Required initial and maintenance margin levels for each commodity.
- Grades and location designations for physical delivery contracts.
- Cash price series (used for final settlement of cash-settled contracts).
- Pricing conventions and minimum price fluctuations.
- Supervision of day-to-day activities of all trading participants.
- Market surveillance (prevents manipulation and congestion).
- Real-time distribution of price data.

Exchanges and Clearinghouses

LO 31.2: Explain the organization and administration of an exchange and clearinghouse.

Futures exchanges provide an orderly market for trading in futures and options on futures. Only individuals can obtain exchange membership and corresponding trading privileges. With exchange approval, individuals may grant some membership privileges to an affiliated person, organization, or corporation.

Generally, a membership-elected Board of Directors oversees the exchange. Day-to-day administrative oversight of the exchange is the responsibility of a full-time, paid, non-member president. The president carries out the policies approved by the Board. Exchange policies start out as a product of several member committees, which then make policy recommendations to the Board, which can either accept or reject the recommendations.

Day-to-day operations, such as legal, compliance, data processing, research, and facilities maintenance are generally handled by the exchange staff. Revenue is generated by transaction fees, assessments, dues paid by exchange members, and fees for access to exchange data. This revenue is used to pay for operating expenses incurred by the exchange.

In the United States, every futures exchange has an affiliation with a **clearinghouse** (also known as a clearing association). The clearinghouse's primary roles are to provide financial mechanisms and guarantee performance on the exchange's futures and options contracts. The clearinghouse acts as a counterparty by substituting itself for each counterparty for the purpose of settling gains and losses. It pays out funds to those receiving profits and collects funds from those with losses. This counterparty substitution is known as the **principle of substitution**. The clearinghouse also simplifies physical deliveries by matching the parties that are taking and making delivery.

The clearinghouse for most U.S. commodity exchanges is organized as a separate member corporation. Other exchanges, such as the Chicago Mercantile Exchange (CME) and the New York Mercantile Exchange (NYMEX), are structured so that the clearinghouse is recognized as a department within the exchange. Note that not all exchange members are clearinghouse members, but all clearinghouse members must be exchange members.

A Board of Directors or a committee of the exchange manages clearinghouse functions. The Board is responsible for setting policy, authorizing the admission or expulsion of members, electing clearinghouse officers, and, on some exchanges, appointing members to an oversight committee. Clearinghouse membership is carefully screened with strict standards for financial strength, administrative ability, and integrity.

Revenue for clearinghouse operations is earned from fees charged for clearing trades, interest from invested capital, temporary investment of member guaranty deposits, and by providing other services such as the handling of delivery notices.

Exchange Membership

LO 31.3: Describe exchange membership, the different types of exchange members, and the exchange rules for member trading.

Individuals who participate in exchange trading must be exchange members or possess trading privileges on the exchange. With regard to an open outcry (i.e., non-electronic) exchange system, key exchange members are as follows:

- **Local:** uses personal capital to trade for a personal account. Relies on short-term trading skills and market analysis. The local may use as many different trading styles as other traders.
- **Scalper:** trades frequently in an attempt to make money on small price movements.
- **Day trader:** trades frequently throughout the day, but has a zero (flat) net position at the end of the day.
- **Floor broker:** trades for others and earns a commission.

These exchange members can complete a transaction after making an open, competitive outcry of the bid or offer in hand. Due diligence is required by the floor broker when executing orders. Losses resulting from broker error are the broker's personal liability. Bids or offers not made openly in the pit (i.e., non-competitive trades) are typically considered invalid trades. Members participating in a trade that fails to conform to exchange rules may be subject to disciplinary action.

Dual trading is the practice of members trading both for their own account and for customer accounts. Rules regarding dual trading usually dictate that customer orders have priority over member orders.

Clearinghouse Functions

LO 31.4: Explain original and variation margin, daily settlement, the guaranty deposit, and the clearing process.

Original and Variation Margin

Member firms are required to deposit funds to support open contracts submitted for clearance, which is also known as **original margin**. The amount (per contract) of original margin is determined by the clearinghouse board of directors. The Commodity Futures Trading Commission (CFTC) regulates that client funds be segregated from a member firm's proprietary funds. That is, clearinghouse members are required to maintain one account for their own proprietary funds and a separate account for client funds.

The historical volatility of futures prices are often used to set margin levels. The levels are used to protect the clearinghouse against a one-day large price movement. Spot month positions (i.e., those positions near delivery) may have different margin levels than hedge positions.

Acceptable original margin may be different for each clearinghouse. Generally, clearinghouse members may submit letters of credit, cash, government securities, or registered securities to satisfy margin requirements. Margin deposits for customers may also include other negotiable instruments, such as warehouse receipts. Contracts priced in a foreign currency may have margin payments denominated in that currency.

Margin may be collected by the clearinghouse on either a gross or net position basis for each customer. Gross original margin is required for each short contract and each long contract on the CME and the NYMEX. However, most clearinghouses permit a member firm to net a customer's open long and open short positions for a particular delivery month of a futures contract. This is accomplished by looking at the total risk exposure and requiring the client to deposit only the margin needed to support the net position. For example, if a clearinghouse member were long 1,000 contracts of July wheat and short 900 contracts of July wheat, a clearinghouse operating on a net basis would only charge margin on the net long 100 contracts. If the clearinghouse collected margin on a gross basis, it would charge margin on 1,900 contracts.

For actively traded delivery months, settlement prices usually fall within the range of prices traded at the close. The closing bid and offer are usually averaged to reach the settlement price. For inactive months that may not have much activity at the close, a nominal price may be designated by an exchange member committee. A nominal price is not an actual traded price but a fabricated price based on the closing prices of similar contracts that traded on the closing day in question.

Variation margin is the settlement of daily gains and losses between member firms. It is settled at the end of each day. An exchange committee, often the committee on quotations, determines the settlement price (i.e., the closing value) for each contract for that trading day. Clearinghouse member firms are required to pay to, or receive from, the clearinghouse the difference between the current settlement price and the trade price for that day *or* the previous day's settlement price for an existing position. Variation margin for member firm positions is collected separately from client account positions. The margin is collected/paid by automatic debit or credit before the opening of trading on the following business day to the member firm's house account or the customer's margin account. Some markets collect margin intraday based on the previous day's open positions. In this case, payment is required within one hour of the intraday margin call.

Guaranty Deposit

Clearinghouse members also must maintain a large **guaranty deposit** with the clearinghouse in addition to original and variation margin in order to maintain their own and customer positions. The guaranty deposit, or reserve, must be maintained with the clearinghouse as long as the firm is a member of the clearinghouse. The deposit can be made with cash, securities, or letters of credit. The clearinghouse has access to the funds at all times to meet the financial needs of any defaulting member.

The responsibility for defaulting individual client accounts falls with the carrying member firm, using its own funds if delivery of funds or a commodity is not satisfied. As a result, the commodity account agreement gives the broker the right to liquidate any customer position, without recourse, in the event that margin calls are not met. Brokers usually insist that margin calls are satisfied promptly because they are responsible for any deficit.

Clearing

Matching trade data submitted by clearinghouse members is a primary activity of the clearinghouse. Members provide ongoing data on trades executed on behalf of their customers and on their own proprietary trading accounts throughout the day to the clearinghouse. The clearinghouse must have confirming records of a trade both from the buying and selling clearinghouse members before it can insert itself as the opposite party to a trade (i.e., the counterparty).

The clearinghouse does not assume the legal and financial obligation of the other party to the trade until the accuracy of all reported transactions has been verified and the original margin has been deposited. The clearinghouse becomes the buyer for anyone who has sold a contract and the seller for anyone who has bought a contract. Acting as a counterparty to every trade allows traders to close out positions even when the original counterparty is not ready to close out its own position. Individual traders rely on the clearinghouse to relieve them of their market obligations when an offsetting transaction takes place for an identical futures contract.

Offsetting transactions (i.e., opening and closing transactions) on the clearinghouse's books allows traders to move freely in and out of the futures markets without any obligation to the original party involved. The difference between the purchase price and the sale price represents the trader's gross profit or loss, before the broker's commission is paid.

LO 31.5: Summarize the steps that are taken when a clearinghouse member is unable to meet its financial obligations on its open contracts.

The following procedures are in place if a clearinghouse member is unable to meet its open contract obligations:

1. A solvent clearinghouse member takes possession of all open fully margined customer positions. All under-margined customer positions and the firm's proprietary positions are liquidated.

2. If the member's customer account with the clearinghouse is in deficit because of a liquidation, any additional margin the member had deposited at the clearinghouse is applied toward the deficit on the customer's positions.

3. If the defunct member's margin deposits on hand are not sufficient to cover the deficit, the member's exchange membership may be sold and the funds deposited in the guaranty fund by the member may be used.

4. If a deficit still exists, the surplus fund of the clearinghouse may be used at the discretion of the clearinghouse board.

5. Contributions by other solvent clearinghouse members to the guaranty fund may also be used. The guaranty fund may be replenished by a pro rata special assessment made by the remaining members of the clearinghouse.

The financial integrity of each futures contract is strengthened by:

- The selection criteria of clearinghouse members.
- The guaranty fund.
- The ability of the clearinghouse to assess members' ability to fulfill obligations.

Because of these safeguards, failure of a clearinghouse member has never resulted in the failure of a U.S. clearinghouse to meet its financial obligations.

Futures Delivery Mechanics

LO 31.6: Describe the mechanics of futures delivery and the roles of the clearinghouse, buyers, and sellers in this process.

The settlement of futures contracts that are not offset by the end of the day is satisfied by delivery or cash settlement. Actual delivery is the physical transfer of a commodity or financial instrument. Financial fulfillment of open futures contracts is guaranteed by the clearinghouse, but physical delivery is not. The clearinghouse member firm guarantees the physical delivery. If a customer defaults, it is up to the member firm to make the receiving client whole.

Physical delivery for some futures and options on futures is not practical and sometimes not possible. For example, futures based on Eurodollar interest rates or stock indices are settled by cash payments rather than delivery of a time deposit or basket of securities. Cash-settled contracts are settled by the buyer or seller paying or receiving the difference between the trade price and the spot price of the underlying instrument or index to the clearinghouse at maturity.

Physical delivery is made easier by having the clearinghouse as the counterparty on every trade. Direct deliveries can be made by a short to a long even though the two parties never actually trade with one another. The clearinghouse receives delivery notices from sellers (shorts) and assigns the notices to buyers (longs). In addition, clearinghouses may also arrange for the inspection of commodities, act as a depository for warehouse receipts, or receive delivery of foreign currencies and some financial instruments.

A **seller's option** gives the seller (short) of a futures contact the right to designate the day when physical delivery is to be made if more than one day is allowed by the contract. The seller can also designate the location or warehouse where delivery will be made. The buyer (long) has no say in the specific day, location, or grade of the commodity or financial instrument being delivered.

When the seller is ready to make delivery, the seller instructs the broker to submit a **notice of intention to deliver** to the clearinghouse. The broker receives a fee for carrying the account. Delivery procedures can vary with contracts and exchanges. A notice of intent to deliver may be submitted from one day to several weeks in advance; the CBOT is submitted two business days prior to delivery. The notice contains all information regarding delivery including grade, weight, place, date, and price.

Once delivery is made, the clearinghouse decides which buyer should receive the delivery. The three methods generally used for assignment are:

- To the member that has the oldest long position open (used by the CBOT).
- To the member with the largest net long position open (on a net basis).
- To the member with the largest gross long position open.

With any of these methods, the firm receiving the notice may allocate the notice to a long customer position. The buyer and seller are brought together by the clearinghouse by exchanging the names of the deliverer and receiver with the two clearing firms involved. The firms involved get the respective parties to complete delivery and settlement directly or by handling the exchange via a clearinghouse. If the exchange is handled by a clearinghouse:

- The clearing member of the delivery customer must provide the required documents, along with a bill for the amount due.
- Once the clearinghouse receives payment by certified check, the clearinghouse will release the documents to the receiver's carrying firm.
- The procedure must be completed within the time span specified by exchange rules.
- The clearinghouse provides the certified check to the delivering customer's firm to provide to its customer.

Open positions can be offset up to and including the last trading day of a futures contract. Positions remaining open beyond that date must be settled by actual delivery or cash settlement, depending on exchange procedures. The **tender day** is the last day on which notices of intention to make delivery may be issued. The last trading day and the tender day are often the same, which means that at the end of trading all shorts have either been covered with offsetting positions or issued delivery notices. All longs either have been offset or are in a position to receive delivery. The last step is the exchange of delivery documents and certified checks.

The sequence of delivery for the three types of delivery procedures are as follows:

Physical Delivery with Concurrent Trading and Delivery

- First notice day:
  - Shorts may, but are not required to, issue a delivery notice on this day or subsequent days, according to the terms of the contract.
  - Trading continues along with deliveries; shorts may issue notices, and longs are liable for delivery.
- Last notice day: last day for delivery notices to be sent.
- Last trading day: last day for trading the contract.

Physical Delivery Following the End of Trading

- Last trading day: last day to trade contracts; remaining open positions are to be satisfied by delivery.
- First notice day: shorts may, but are not required to, issue a delivery notice on this day or subsequent days, according to the terms of the contract.
- Last notice day: last day a short can issue a delivery notice.

Cash Settlement

- Last trading day: last day to trade contracts; remaining open contracts are settled in cash after trading is over.
- Cash settlement day: all remaining open positions are settled by cash debit or credit using a cash-market value; must be done by exchange-specified date and time.

Speculators have little interest in taking delivery of a commodity. If a delivery notice is received, the speculator would be responsible for all associated expenses of delivery and ownership, such as carrying broker fees, interest expense, storage, insurance, additional inspections (if resold), and possibly reselling at a discount if the commodity fails inspection. One option for the long speculator is to sell his position in the spot month and buy the same number of contracts for a later month. This is known as switching or rolling over a futures position. The extra cost of commission and transaction fees is less expensive than taking ownership.

Futures Market Participants

LO 31.7: Explain the role of futures commission merchants, introducing brokers, account executives, commodity trading advisors, commodity pool operators, and customers.

Futures Commission Merchants

Futures commission merchants (FCMs) are futures brokerage firms that are designated by the Commodity Exchange Act to operate as intermediaries between public customers, including hedgers and institutional investors, and the exchanges. An FCM is also known as a **commission house** or **carrying firm**. An FCM has the following characteristics:

- Provides facilities to execute customer orders.
- Maintains records of each customer's open position, margin balance, and transactions.
- Earns a commission for the services it provides.
- Only entity other than the clearinghouse that can hold customer funds.
- May be full service or discount firm.
- May be a national or regional brokerage company offering other financial services, or may just offer futures or options on futures.
- May have a parent company or related company in agribusiness, commercial banking, or other commercial enterprise.

Introducing Brokers

An individual or firm that has established relationships with one or more futures brokerage firms is an introducing broker (IB). An IB is similar to an FCM in that it is responsible for maintaining and servicing customer accounts and relationships, and like FCMs, its sales force receives commissions. Unlike FCMs, IBs cannot hold customer funds, so, as a result, all IB customers must maintain accounts with an FCM where each customer account is identified and carried separately on the books of an FCM (on a fully disclosed basis).

There are two types of introducing brokerage firms: independent and guaranteed. Independent IBs operate independently of any particular brokerage firm. They often have sufficient capital to meet regulatory requirements and choose to introduce their clients through different FCMs. This contrasts with a guaranteed IB that has a legal and regulatory relationship with the guarantor futures commission merchant through which the IB introduces its customers. FCMs and IBs must be registered under the Commodity Exchange Act.

Account Executives

The account executives of the FCMs and IBs must be registered as **associated persons** (APs). They are agents of the FCM or IB and deal directly with the firm's customers. Account executives are paid based on the commissions their clients pay to the firm where they are employed. The role of account executives is to:

- Provide the appropriate documents for new accounts and confirm they are signed.
- Clarify trading rules, disclosure requirements, and procedures to customers.
- Inform clients of prices and market conditions.
- Enter orders for clients.
- Convey executed prices of trades and pending order statuses.
- Act as liaison between the customer and the firm's research department.
- Notify customers of margin calls.
- Establish and maintain customer accounts with current information.
- Identify their customer according to *National Futures Association (NFA) Rule 2-30: Customer Information and Risk Disclosure*. This rule calls for FCMs to know their customers by obtaining (1) name, address, and occupation, (2) estimated annual income and net worth, (3) approximate age, and (4) previous investment and futures trading experience.

The account executive obtains client information to conduct due diligence on the client's financial means and relevant investment experience in order to ensure that futures contracts are an appropriate investment vehicle.

Account executives have traditionally been relied upon to be the client's point of contact, collect margin funds, and ensure clients are trading within their means. The account executive, as a relationship manager, helps the FCM avoid compliance or bad debt issues related to customer accounts. However, note that customers of an FCM are now able to forgo contact with account executives due to advancements in electronic trading and communication.

The solicitation and marketing of clients is left to research or marketing departments rather than account executives. The rules covering such content are covered under *NFA Rule 2-29: Promotional Material and Communication with the Public*. As a result, of the NFA's strict rules, most firms do not allow account executives to write a personal commodity advisory or marketing letter for distribution to existing customers.

Commodity Trading Advisors

The two broad categories of money managers in the futures and options industry are commodity trading advisors (CTAs) and commodity pool operators (CPOs). Both are

regulated by the CFTC and the NFA. CTAs trade individual accounts for single clients, while CPOs pool the funds of many investors and trade for all individuals pooled under one account. Firms that cater services to large corporate or institutional investors play the role of manager of managers, matching CTAs with a specific client investment objective and risk tolerance. A group of CTAs may also be assembled to meet the needs of a large client and may register as either a CPO or a CTA.

A CTA is defined by the Commodity Exchange Act as an individual or organization that, for compensation, advises others on trading futures and options on futures. Characteristics of a CTA include:

- Similar to brokerage accounts, decisions regarding what and when to trade are made by a professional trading advisor with discretion over trading.
- The power of attorney gives the CTA discretionary authority.
- Trading philosophy should achieve investor's goals (based on whether risk/return profile will be satisfied by the fundamental or technical analysis used).
- Can trade and manage funds controlled by CPOs.
- If CTAs are also FCMs, they may accept funds from customers in the CTA's name.

Service fees charged by CTAs include:

- *Performance (trading) fee*: usually based on cumulative profits above a certain level at the end of each quarter.
- *Management fee*: usually a yearly percentage of assets under management, paid monthly or quarterly whether the account makes or loses money.
- *Brokerage commissions*: not all CTAs participate, but if commissions are shared with its FCM, it must be disclosed to the CTA's clients.

Commodity Pool Operators

A CPO is defined by the Commodity Exchange Act as a syndicate-type business that solicits and pools funds, securities, or property for the purpose of trading in futures. Similar to a securities industry mutual fund, they combine funds of a number of investors to trade futures and options. Benefits of CPOs include:

- Professional management of client funds for as little as $5,000. Individually managed accounts may have minimums of more than $50,000.
- Allows access to different CTAs with different styles and systems, offering diversification.
- CPOs are usually the general partners of a limited partnership offering limited liability for investors. The pool offers flow-through taxation benefits and limits investor risk to the capital invested. Investors in individual commodity accounts can lose more than they initially placed in the account.
- CPOs can have dissolution clauses. For example, a unit with a 75% dissolution clause and an initial value of $1,000 would stop trading when its value reached $250 and the investor would receive close to 25% of his money back.

Customers

Customers of an FCM include:

- Firms or companies wishing to hedge.
- Individual speculative traders.
- Money management firms, money managers, and institutional investors.

- Floor brokers not belonging to the clearinghouse, wishing to use the clearing facilities of the commission house.
- Brokerage firms that are not members of a particular exchange or clearinghouse.

Futures transactions made with one commission house by another may be carried on the books of the clearing firm under individual names of the other firm's customers (i.e., a disclosed basis) or in an omnibus account (i.e., one large account containing all the trades and positions of the firm's customers). With an omnibus account, it is up to the originating firm to keep detailed accounting records for each individual customer.

Customer margin funds deposited with the firm are subject to special rules. Customer funds received by the firm designated to fund open margin positions cannot be commingled with funds belonging to the commission house itself. Customer margin funds plus all related realized and unrealized profits from futures positions must be deposited into a bank account separate from the firm's own funds. These safeguards are in place so that if the commission house business should fail, the residual margin funds and profits cannot be used to satisfy general creditors. The funds are available only for the firm's futures customers or used by the commission house to meet the clearinghouse's margin requirements for the firm's customer base.

Key Concepts

LO 31.1

Futures exchanges differ from stock exchanges in that futures exchanges author all terms and conditions of standardized contracts. Common contract terms and trading rules include:

- Contact/delivery months available for trading.
- Initial and maintenance margin levels for each contract.
- Grades and location designation for physical delivery contracts.
- Cash price series for final settlement of cash-settled contracts.
- Pricing conventions and minimum price fluctuations.
- Supervision of day-to-day activities of all participants involved in trading.
- Market surveillance to prevent manipulation and congestion.
- Real-time distribution of price data.

LO 31.2

Exchange membership and the corresponding trading privileges are only for individuals. With exchange approval, individuals may grant some membership privileges to an affiliated person, organization, or corporation.

In the United States, every futures exchange has an affiliation with a clearinghouse or clearing association. The clearinghouse's primary role is to provide financial mechanisms and to guarantee performance on the exchange's futures and options contracts. The clearinghouse operates under the principle of substitution, substituting itself for each counterparty for the purpose of settling gains and losses, paying out funds to those receiving profits, and collecting funds from those with losses.

The clearinghouse for most U.S. commodity exchanges is organized as a separate member corporation, while others have the clearinghouse as a department within the exchange. Not all exchange members are clearinghouse members, but all clearinghouse members have to be exchange members.

LO 31.3

The open outcry (i.e., non-electronic) exchange system has several key participants:

- Locals use their own capital to trade for their own account, relying on short-term trading skills and market analysis.
- Scalpers trade frequently in an effort to make money on small price movements.
- Day traders trade frequently throughout the day, but have a zero (flat) net position at the end of the day.
- Floor brokers trade for others and earn a commission.

LO 31.4

Member firms are required to deposit funds to support open contracts submitted for clearance, which is also known as original margin. The amount per contract of original margin is determined by the clearinghouse board of directors.

Clearinghouse member firms are also required to pay to, or receive from, the clearinghouse the difference between the current settlement price and the trade price for a given day *or* the previous day's settlement price for an existing position. The difference is the variation margin. The margin is collected/paid by automatic debit or credit before the opening of trading on the following business day to the member firm's house account or the customer's margin account.

Clearinghouse members must maintain a large guaranty deposit with the clearinghouse. The clearinghouse has access to the funds at all times to meet the financial needs of any defaulting member.

LO 31.5

The following procedures are in place if a clearinghouse member is unable to meet its open contract obligations:

- A solvent clearinghouse member takes possession of all open fully-margined customer positions. All under-margined customer positions and the firm's proprietary positions are liquidated.
- If the member's customer account with the clearinghouse is in deficit because of liquidation, any additional margin the member had deposited at the clearinghouse is applied toward the deficit.
- If a deficit still exists, the member's exchange membership may be sold and the funds deposited in the guaranty fund by the member may be used.
- If a deficit still exists, the surplus fund of the clearinghouse may be used.
- Contributions by other solvent clearinghouse members to the guaranty fund may also be used.

LO 31.6

The settlement of futures contracts not offset by the end of the day is satisfied by actual delivery or cash settlement. Physical delivery is made easier by having the clearinghouse as the counterparty on every trade. Direct deliveries can be made by a short to a long even though the two parties never actually trade with one another. The clearinghouse receives delivery notices from sellers (shorts) and assigns the notices to buyers (longs).

LO 31.7

Futures commission merchants (FCMs) are futures brokerage firms that are designated by the Commodity Exchange Act to act as intermediaries between public customers, including hedgers and institutional investors, and the exchanges.

Introducing brokers (IB) consist of individuals and firms that have established relationships with futures brokerage firms.

The account executives of the FCMs and IBs must be registered as associated persons. They are agents of the FCM or IB and deal directly with the firm's customers.

The two broad categories of money managers in the futures and options industry are commodity trading advisors (CTAs) and commodity pool operators (CPOs). CTAs trade individual accounts for single clients and CPOs pool the funds of many investors and trade for all the individuals pooled into one account.

Concept Checkers

1. With regard to futures trading, which of the following statements best describe the principle of substitution?
   A. The clearinghouse changes the designated currency before settlement.
   B. The exchange takes on the role of the counterparty.
   C. The clearinghouse takes on the role of the counterparty.
   D. The exchange can substitute cash for a given commodity at its discretion.

2. XYZ, a clearinghouse member, has recently contributed funds with its clearinghouse. The funds are designed to give the clearinghouse the ability to meet the financial obligations of any defaulting members. The funds may not be withdrawn by XYZ as long as it remains a member of the clearinghouse. Which of the following amounts best describe XYZ's contribution?
   A. Variation margin.
   B. Original margin.
   C. Membership dues.
   D. Guaranty deposit.

3. ABC, a clearinghouse member, has not managed its debts very well. As a result, it is unable to meet its open contract obligations. Which of the following statements represents one of the first actions of the clearinghouse?
   A. Exchange membership is sold.
   B. Under-margined customer positions are transferred to a solvent clearinghouse member.
   C. Guaranty fund is used.
   D. Fully margined positions are transferred to a solvent clearinghouse member.

4. Jack Johnson is going to receive a physical commodity from a settling long futures trade. Which of the following statements best describe the role of Johnson and the clearinghouse in this process?
   A. The clearinghouse will coordinate Johnson's settlement with any eligible settling shorts.
   B. Johnson will have to contact the clearinghouse to coordinate with any eligible settling short.
   C. Johnson will have to close his position with the original counterparty.
   D. The clearinghouse will coordinate Johnson's settlement with the original counterparty only.

5. Which of the following statements best describe the difference between a commodity trading advisor (CTA) and a commodity pool operator (CPO)?
   A. CTAs trade for single clients with individual accounts and CPOs trade for pooled funds.
   B. The only difference relates to the fees charged.
   C. Only CPOs must be registered as associated persons.
   D. CTAs can only trade futures, while CPOs can trade futures and options.

Concept Checker Answers

1. C The clearinghouse's primary role is to provide financial mechanisms and to guarantee performance on the exchange's futures and options contracts. The clearinghouse operates under the principle of substitution, substituting itself for each counterparty for the purpose of settling gains and losses, paying out funds to those receiving profits, and collecting funds from those with losses.

2. D Clearinghouse members are required to provide not only original and variation margin to maintain their own and customer positions, but also must maintain a large guaranty deposit with the clearinghouse. The deposit, or reserve, must be maintained with the clearinghouse as long as the firm is a member of the clearinghouse. The deposit can be made with cash, securities, or letters of credit. The clearinghouse has access to the funds at all times to meet the financial needs of any defaulting member.

3. D The first action of the clearinghouse is to move fully margined customer positions to a solvent clearinghouse member.

4. A Futures market physical delivery is made easier by having the clearinghouse as the counterparty on every trade. Direct deliveries can be made by a short to a long even though the two parties never actually trade with one another. The clearinghouse receives delivery notices from sellers (shorts) and assigns the notices to buyers (longs).

5. A CTAs trade individual accounts for single clients. CPOs pool the funds of many investors and trade for all individuals pooled under one account.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Hedging with Futures and Options

Topic 32

Exam Focus

Futures and options on futures are frequently used by hedgers to manage price risk. However, these instruments possess basis risk, which can significantly damage a hedger's portfolio. For the exam, understand terminology related to the basis, including what it means to be long or short the basis. In addition, understand how exchange for physical transactions are conducted. Finally, know how options on futures are used as hedging instruments.

The Basics of Hedging

Despite their link to physical assets, futures and options are rarely used to actually acquire underlying assets. Physical delivery rarely occurs with futures contracts primarily because the contracts are mismatched in desired quality, location, and timing relative to a hedger's preferences. Instead, futures are primarily used as a means to allow market participants to control the price risk associated with the underlying asset.

Hedgers in the futures market should seek contracts that closely align with the characteristics of the underlying asset to minimize the risk associated with a mismatched hedge. Fortunately, price correlation between the futures price and the cash (i.e., spot) price of a commodity tends to be high. This characteristic is essential in order for futures to be an effective part of a hedging or speculation strategy. Later in this topic, we will examine how this relationship is measured using basis and its implications for market participants.

Hedges that use futures to manage price risk on cash positions can be thought of in either of the following ways.

- Futures can be used in place of a cash transaction that would occur at a later date (i.e., serve as a temporary substitute).
- Futures contracts and cash positions can be considered equal and opposite positions. For example, consider an individual who holds a long position in a gold exchange-traded fund (ETF). The long position in this ETF would be considered a cash position, and the opposite futures position would be a short futures position.

Note that these two approaches result in similar conclusions when analyzing a hedger's risk management activities. Figure 1 explains how futures contracts can be applied in common hedging situations.

Figure 1: Hedging with Futures

Situation	*Hedging Strategy*
Individual who owns or will soon own the underlying asset and wants to lock in the price (e.g., a farmer who will harvest corn in three months).	Sell futures contracts now and offset this position at a later date with the sale of the physical asset in the cash market.
Individual who needs to acquire the underlying asset in the future and wants to lock in the price.	Buy futures contracts now and either accept delivery or purchase in the cash market at a later date and offset the futures position (more efficient than taking delivery).

Basis

LO 32.1: Define the terms "long the basis" and "short the basis."

Basis is defined as the cash price less the futures price. In equation form, basis is represented as follows:

$$\text{basis} = S_t - F_0$$

where:
S_t = cash (or spot) price of the underlying asset at time t
F_0 = current price of the futures contract

Given this equation, it is clear that *positive basis* (i.e., *cash over futures*) occurs when the cash price exceeds the futures price and *negative basis* (i.e., *cash under futures*) occurs when the futures price exceeds the cash price.

While futures and options reduce price risk for those holding cash positions in an asset, futures positions will contain **basis risk**. When hedging with futures, a potential change in the basis is unavoidable, and this change can work either for or against a hedger. However, in hypothetical hedging examples, the basis is often assumed to be stable during the hedging period to allow for a clearer illustration of the mechanics of hedging.

Using the previously stated definition of basis, hedgers who utilize futures can choose a set of positions that are either long the basis or short the basis.

Long the basis refers to a set of positions that consists of a short futures position and a long cash position. Positions that are long the basis benefit when the basis is *strengthening*. This occurs when the cash price increases at a faster rate than the futures price *or* when the cash price decreases at a rate slower than the futures price. Note that a position that is long the basis is equivalent to a selling hedge. A *selling hedge* (or *short hedge*) occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long cash position. Therefore, a selling hedge, or being long the basis, is appropriate when a hedger possesses inventory of an asset and expects prices to decline.

Example: Long the basis

Matt McCoy is buying 40,000 bushels of corn today. Assume the cash price for corn is \$5.00/bushel and the three-month futures contract price is \$5.35. Three months later, the cash price of corn drops to \$4.50, and McCoy sells his physical corn inventory and closes out his short futures position by buying to offset at \$4.50. **Calculate** McCoy's profit/loss at the end of three months if he used three-month futures to fully hedge his physical corn inventory.

Answer:

<u>Cash Position</u>

Today's price: \$5.00/bushel; Buy corn

Price in three months: \$4.50/bushel; Sell corn

Net profit/loss per bushel = \$4.50 – \$5.00 = –\$0.50

<u>Futures Position</u>

Today's price: \$5.35/bushel; Short corn futures

Price in three months: \$4.50/bushel; Buy corn futures to close out futures position

Net profit/loss per bushel = \$5.35 – \$4.50 = \$0.85

Total net profit/loss = (\$0.85 – \$0.50) × 40,000 = \$14,000

In this example, McCoy is long the basis because he is long the cash position and short the futures position. While McCoy suffered a loss on the cash position, the profits on the short futures position exceeded those losses. Thus, since the cash price deceased by 10% and the futures price decreased by 16%, McCoy earned a significant profit.

Professor's Note: Being long the basis can also be classified as a cash and carry hedge.

Short the basis refers to a set of positions that consists of a long futures position and a short cash position. Positions that are short the basis benefit when the basis is *weakening*. This occurs when the futures price increases at a faster rate than the cash price *or* when the futures price decreases at a rate slower than the cash price. Note that a position that is short the basis is equivalent to a buying hedge. A *buying hedge* (or *long hedge*) occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. Therefore, a buying hedge, or being short the basis, is appropriate when a hedger needs to acquire inventory at some point in the future and expects prices to rise.

Example: Short the basis

WRE, Inc., forecasts that copper prices will rise and has been told that a short the basis hedging strategy would allow them to profit from this movement if their forecast is correct. Assume the cash price for copper is $3.25/pound and the six-month futures contract price is $0.40 under the cash price. Three months later, the cash price of copper increases to $3.35, and the three-month futures price is $3.10. Calculate WRE's profit/loss at the end of three months if it used six-month futures and 10,000 pounds of copper to execute a short the basis strategy.

Answer:

Cash Position

Today's price: $3.25/pound; Sell copper

Price in three months: $3.35/pound; Buy copper

Net profit/loss per pound = $3.25 - $3.35= -$0.10

Futures Position

Today's price: $2.85/pound; Buy copper futures

Price in three months: $3.10/pound; Sell copper futures to close out futures position

Net profit/loss per pound = $3.10 – $2.85 = $0.25

Total net profit/loss = (–$0.10 + $0.25) × 10,000 = $1,500

In this example, WRE is initially short the cash position and long the futures position in order to be short the basis. During the hedging period, the basis weakened, so WRE was able to earn a profit.

Exchange for Physical Transactions

LO 32.2: Explain exchange for physical (EFP) transactions and their role in the energy and financial futures markets.

An **exchange of futures for physicals**, or simply, *exchange for physicals* (EFPs) is an off-exchange, private transaction between two parties where one party purchases the physical asset and sells a futures contract on that asset to a counterparty while the counterparty sells the physical asset and buys the futures contract. Prior to the transaction, the parties agree on the futures price, the cash price, the quantity, delivery date, and commodity grade (when applicable). After the transaction is complete, it must be reported to the appropriate clearinghouse. An EFP transaction is the one exception to the U.S. Federal law that requires

that all trades take place on the floor of the exchange (i.e., cannot be prearranged). This allows EFP transactions to occur during periods when the exchange is closed.

An EFP transaction allows both parties to easily close their hedge at the expiration of the agreement while also allowing them to remove any price or counterparty variability because prices are fixed throughout the term of the transaction. The utilization of EFPs tends to be restricted only to traders that actually trade the underlying cash asset. Note that the customization and private nature of an EFP transaction stands in stark contrast to the standardization of public futures contracts.

EFPs are frequently used in the energy and financial futures markets. In the energy market, the futures price used is often quoted in terms of the basis differential. The differential is the price difference that is attributable to differences in the characteristics of the physical asset and the product specified in the futures contract. For example, a cash position in sweet crude oil and a futures position in sour crude oil would require a differential to be specified in the EFP transaction as the price correlation between the two assets is not perfect. In the financial futures market, market participants such as currency dealers, trading desks, hedgers, and speculators use EFPs to establish or offset hedges and improve transactional efficiency. For example, a given market may lack the necessary depth for a large position to be established at a single price and a participant who wishes to take a large position will encounter price slippage (i.e., a difference between expected transaction costs and the actual amount paid). By using an EFP transaction, the participant can avoid this issue.

Options on Futures

LO 32.3: Outline and calculate the payoffs on the various scenarios for hedging with options on futures.

Hedgers may also choose to use options on futures rather than futures contracts themselves to hedge price risk in a given market. **Options on futures** give the holder the right to buy or sell a specified futures contract on or before a given date at a given futures price (i.e., the strike price). Similar to futures, a hedger would select options in the amount and type that allow for the elimination of the risk present in their long or short cash position (i.e., take a position in options on futures that is equal and opposite their cash position).

Professor's Note: When hedging with options on futures, there is no such thing as a long hedge or a short hedge.

Options on futures are more complex than futures because they involve an additional relationship. That is, options on futures are priced based on the relationship between (1) the cash price and the futures price and (2) the futures price and the option price. To better illustrate this financial derivative, it is important to examine the characteristics of both put and call options on futures and how these options are utilized.

Long Put Options on Futures

Assume an individual has a large amount of inventory. If this individual used futures to hedge price risk, they would take a short futures position. If options on futures were used instead, the individual could purchase put options on futures contracts to hedge the cash position (i.e., take a long put position). The following example illustrates the payoff for this hedging strategy.

Example: Inventory hedging with puts

The AUG Specialty Fund (AUG) currently holds gold in its inventory. However, AUG would like to minimize price risk over the next six months. Assume the following gold prices (on a per ounce basis):

Cash price: $1,200
6-month futures price: $1,300
6-month put with strike price of $1,300: $60

Now assume that in six months (at expiration of the option), the cash price of gold has fallen to $1,100, the futures price has fallen to $1,200, and the put option price is $100. **Calculate** the profit/loss on a per ounce basis if AUG:

1. Does not hedge its cash position.

2. Uses futures to hedge its cash position.

3. Uses long puts on futures to hedge its cash position.

Answer:

1. Unhedged Cash Position

Unhedged profit/loss = ($1,100 – $1,200) = –$100

2. Cash Position Hedged with Futures

Short futures profit/loss = ($1,300 – $1,200) = $100

Net profit/loss = cash profit/loss + futures profit/loss

Net profit/loss = –$100 + $100 = 0

3. Cash Position Hedged with Put Option on Futures

Long put profit/loss = $100 – $60 = $40

Net profit/loss = cash profit/loss + option profit/loss

Net profit/loss = –$100 + $40 = –$60

In this example, the put position reduced the losses on AUG's position, but still resulted in a net loss for AUG despite the fact that the basis remained stable.

Short Call Options on Futures

An individual who wishes to hedge inventory could utilize short call options on futures rather than long put options. The following example illustrates the payoff for this hedging strategy.

Example: Inventory hedging with calls

The AUG Specialty Fund (AUG) currently holds gold in its inventory. However, AUG would like to minimize price risk over the next six months. Assume the following gold prices (on a per ounce basis):

Cash price: $1,200
6-month futures price: $1,300
6-month call with strike price of $1,300: $50

Now assume that in six months (at expiration of the option), the cash price of gold has fallen to $1,100, the futures price has fallen to $1,200, and the call option price is $0. Calculate the profit/loss on a per ounce basis if AUG uses short calls on futures to hedge its cash position.

Answer:

Cash Position Hedged with Call Option on Futures

Short call profit/loss = \$50 – \$0 = \$50

Net profit/loss = cash profit/loss + option profit/loss

Net profit/loss = –\$100 + \$50 = –\$50

In this example, the call position reduced the losses on AUG's position, but still resulted in a net loss for AUG. Note that the short call upside is limited to the amount of the option premium. In this example, the initial premium was \$50.

Professor's Note: These last two examples involved holding options on futures to expiration, so option deltas for the positions were irrelevant. If a hedger wishes to exit their option position prior to expiration, the option delta would provide valuable information about the relationship between the option and the cash price. As you will see in Book 4, a call option delta is between 0 and 1, and a put option delta is between –1 and 0. Option deltas close to zero indicate that an option is out-of-the-money.

Key Concepts

LO 32.1

Long the basis refers to a set of positions that consists of a short futures position and a long cash position. Positions that are long the basis benefit when the basis is strengthening. This occurs when the cash price increases at a faster rate than the futures price or when the cash price decreases at a rate slower than the futures price.

Short the basis refers to a set of positions that consists of a long futures position and a short cash position. Positions that are short the basis benefit when the basis is weakening. This occurs when the futures price increases at a faster rate than the cash price or when the futures price decreases at a rate slower than the cash price.

LO 32.2

An exchange for physicals (EFPs) is an off-exchange transaction where one party purchases the asset and sells a futures contract while the counterparty sells the asset and buys the futures contract. EFPs are frequently used in the energy and financial futures markets, but tend to be restricted only to traders that actually trade the underlying cash asset.

LO 32.3

Options on futures provide the right to buy or sell a specified futures contract on or before a given date at a given futures price (i.e., the strike price). Options on futures are more complex than futures as they are priced based on the relationship between (1) the cash price and the futures price and (2) the futures price and the option price.

Concept Checkers

1. If the futures price for a commodity is less than the spot price of the underlying physical asset, this market structure is termed:
   A. positive basis.
   B. negative basis.
   C. cash under futures.
   D. cash over spot.

2. Brad Oliver is long gold futures and has a short cash position in gold. Which of the following statements regarding Oliver's investments is most accurate? This set of positions:
   A. is long the basis.
   B. is known as a buying hedge.
   C. represents an exchange for physicals (EFPs).
   D. will likely earn a profit when the basis strengthens.

3. Holly Neumann is a soybean farmer who would like to reduce the risk of soybean prices declining before harvest. Which of the following trades would most likely accomplish this objective?
   A. Selling soybean futures.
   B. Purchasing soybean futures.
   C. Purchasing call options on soybean futures.
   D. Purchasing both soybean futures and call options on soybean futures.

4. Which of the following characteristics is not an advantage of an exchange for physicals (EFPs)?
   A. Price certainty.
   B. Standardization.
   C. Privately negotiated.
   D. Can trade at any time.

5. Alex Harrison expects that his company will need 50,000 pounds of copper in three months and plans on using call options on copper futures to hedge the price risk associated with this purchase. Assume the following copper prices:

   Cash price: $3.25/pound
   3-month futures price: $3.30/pound
   Premium on call option with strike price of $3.30: $0.10

   In three months (at expiration of the option), the cash price of copper has increased to $3.50, the futures price has increased to $3.55, and the call option price is $0.25. What is the profit/loss on a per pound basis if Harrison used calls on futures to hedge his company's price risk?
   A. –$0.25.
   B. –$0.10.
   C. $0.15.
   D. $0.25.

Concept Checker Answers

1. **A** Basis is defined as the cash or spot price of the physical asset less the futures price. If the futures price is lower than the spot price, it would be characterized as a market structure with positive basis or cash over futures.

2. **B** A set of positions that involves a long futures position and a short cash position is known as a buying hedge, or long hedge, and is short the basis. There is not enough information provided to conclude that this is an EFP transaction.

3. **A** Neumann should sell futures contracts on soybeans now and offset this position at harvest with the sale of the physical asset in the cash market. This pair of transactions should reduce Neumann's exposure to price risk. Neumann also could have purchased put options on soybean futures to hedge her price risk.

4. **B** An EFPs provides several benefits to its counterparties including price certainty, customization, private negotiations, around the clock availability, and certainty of execution without regard to market depth.

5. **B** Because Harrison is hedging a short cash position, he would need to buy call options on futures.

   profit/loss on cash position: \$3.25 – \$3.50 = –\$0.25

   profit/loss on call options: \$0.25 – \$0.10 = \$0.15

   net profit/loss = –\$0.25 + \$0.15 = –\$0.10

   While the increase in copper prices caused Harrison to incur a net loss, the call option position helped reduce the magnitude of that loss.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Introduction (Options, Futures, and Other Derivatives)

Topic 33

Exam Focus

In this topic, we present the basic concepts of derivative securities and derivative markets. For the exam, know the basic derivative terms as well as the terms related to derivative markets. Also, be able to compute payoffs for the different derivative securities. Finally, be able to create a hedge and know how to take advantage of an arbitrage situation.

Derivative Markets

LO 33.1: Differentiate between an open outcry system and electronic trading.

An **open outcry system** and **electronic trading system** are different forms of trading securities (matching buyers with sellers). The open outcry system (e.g., CBOT) is the more traditional system, which involves traders actually indicating their trades through hand signals and shouting. Electronic trading does not involve an actual "physical" exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ).

LO 33.2: Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages.

An **over-the-counter (OTC) market** differs from a traditional exchange. It is a customized trading market which utilizes telephone and computers to make trades. This market typically involves much larger trades than traditional exchanges. The most typical OTC trade is conducted over the phone. Since terms are not specified by an "exchange," participants have more flexibility to negotiate the most mutually agreeable or attractive trade.

The OTC market is several times the size of the traditional exchange market. For example, in 2007, the OTC market was over $500 trillion, while the exchange-traded market was under $100 trillion.

Advantages of over-the-counter trading:

- Terms are not set by any exchange.
- Participants have flexibility to negotiate.
- In the event of a misunderstanding, calls are recorded.

Disadvantages of over-the-counter trading:

- OTC trading has more credit risk than exchange trading. Exchanges are organized in such a way that credit risk is eliminated.

Basics of Derivative Securities

LO 33.3: Differentiate between options, forwards, and futures contracts.

An **option contract** is a contract that, in exchange for the option price, gives the option buyer the right, but not the obligation, to buy (sell) an asset at the exercise price from (to) the option seller (buyer) within a specified time period, or depending on the type of option, a precise date (i.e., expiration date). A call option gives the option holder the right to purchase the underlying asset by a certain specified date for a specified (in advance) price. A put option gives the option holder the right to sell the underlying asset by a selected date for a pre-selected price.

A **forward contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future. There is no standardization for forward contracts, and these contracts are traded in the over-the-counter market. One party takes the long position, agreeing to purchase the underlying asset at a future date for a specified price, while the other party is the short, agreeing to sell the asset on that same date for that same price. Forward contracts are often used in foreign exchange situations as these contracts can be used to hedge foreign currency risk.

A **futures contract** is a more formalized, legally binding agreement to buy/sell a commodity/financial instrument in a pre-designated month in the future, at a price agreed upon today by the buyer/seller. Futures contracts are highly standardized regarding quality, quantity, delivery time, and location for each specific commodity. These contracts are typically traded on an exchange.

Professor's Note: Remember that a futures contract is an obligation/promise to actually complete a transaction, while an option is simply the right to buy/sell.

LO 33.4: Identify and calculate option and forward contract payoffs.

Call Option Payoff

The payoff on a **call option** to the option buyer is calculated as follows:

$$C_T = \max(0, S_T - X)$$

where:
C_T = payoff on call option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-C_T$ [i.e., $-\max(0, S_T - X)$]. We should note that $\max(0, S_t - X)$, where time, *t*, is between 0 and T, is also the payoff if the owner decides to exercise the call option early (in the case of an American option as we will discuss later).

The price paid for the call option, C_0, is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = C_T - C_0$$

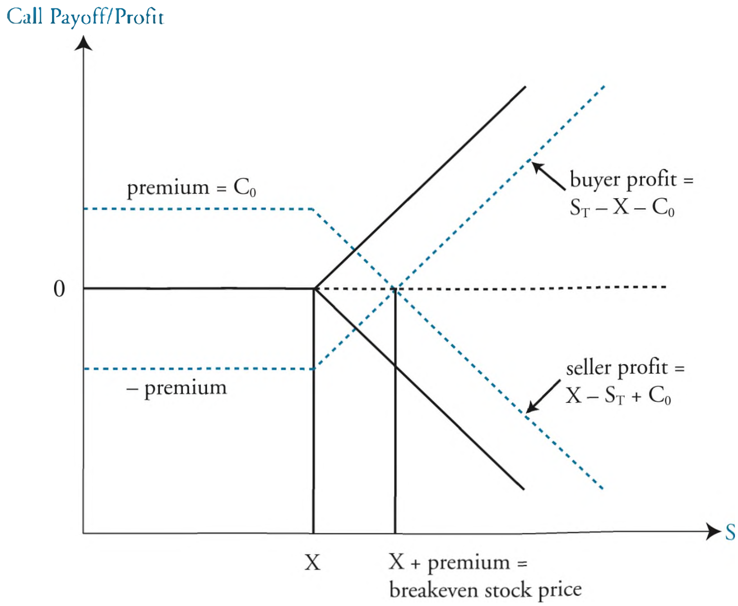
where:
C_T = payoff on call option
C_0 = call premium

Conversely, the profit to the option seller is:

$$\text{profit} = C_0 - C_T$$

Figure 1 depicts the payoff and profit for the buyer and seller of a call option.

Figure 1: Profit Diagram for a Call at Expiration



Put Option Payoff

The payoff on a **put option** is calculated as follows:

$$P_T = \max(0, X - S_T)$$

where:
P_T = payoff on put option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-P_T$ [i.e., $-\max(0, X - S_T)$]. We should note that $\max(0, X - S_t)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option, P_0, is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = P_T - P_0$$

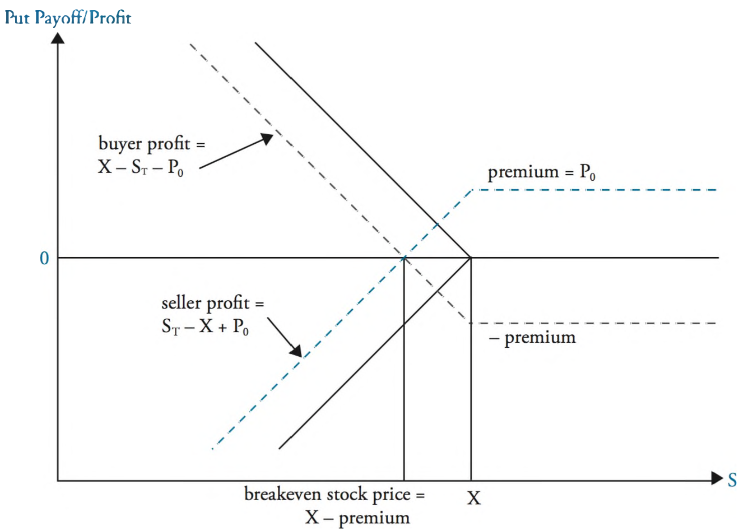
where:
P_T = payoff on put option
P_0 = put premium

The profit to the option seller is:

$$\text{profit} = P_0 - P_T$$

Figure 2 depicts the payoff and profit for the buyer and writer of a put option.

Figure 2: Profit Diagram for a Put at Expiration



Example: Calculating profit and payoffs from options

Compute the payoff and profit to a call buyer, a call writer, put buyer, and put writer if the strike price for both the put and the call is \$45, the stock price is \$50, the call premium is \$3.50, and the put premium is \$2.50.

Answer:

Call buyer:
$\text{payoff} = C_T = \max(0, S_T - X) = \max(0, \$50 - \$45) = \$5$
$\text{profit} = C_T - C_0 = \$5 - \$3.50 = \$1.50$

Call writer:
$\text{payoff} = -C_T = -\max(0, S_T - X) = -\max(0, \$50 - \$45) = -\$5$
$\text{profit} = C_0 - C_T = \$3.50 - \$5 = -\$1.50$

Put buyer:
$\text{payoff} = P_T = \max(0, X - S_T) = \max(0, \$45 - \$50) = \$0$
$\text{profit} = P_T - P_0 = \$0 - \$2.50 = -\$2.50$

Put writer:
$\text{payoff} = -P_T = -\max(0, X - S_T) = -\max(0, \$45 - \$50) = \$0$
$\text{profit} = P_0 - P_T = \$2.50 - \$0 = \$2.50$

Forward Contract Payoff

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

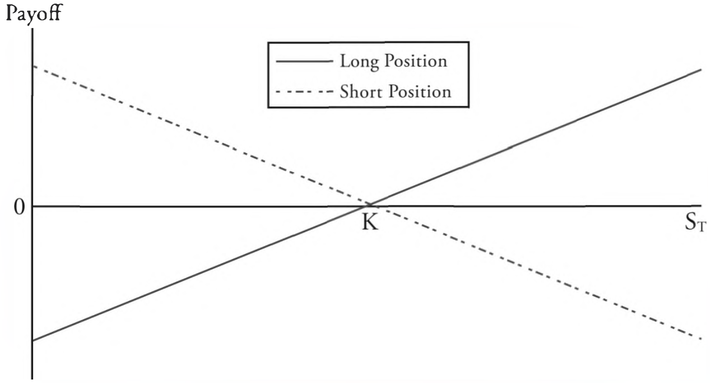
where:
S_T = spot price at maturity
K = delivery price

Conversely, the payoff to a short position in a forward contract is calculated as follows:

$$\text{payoff} = K - S_T$$

Figure 3 depicts the payoff for the long and short positions in a forward contract.

Figure 3: Forward Contract Payoff



Example: Calculating forward contract payoffs

Compute the payoff to the long and short positions in a forward contract given that the forward price is $25 and the spot price at maturity is $30.

Answer:

Payoff to long position:
payoff = $S_T - K$ = \$30 – \$25 = \$5

Payoff to short position:
payoff = $K - S_T$ = \$25 – \$30 = –\$5

Hedging Strategies

LO 33.5: Calculate and compare the payoffs from hedging strategies involving forward contracts and options.

Hedgers use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Hedgers use forward contracts to lock in the price of the underlying security. Forward contracts do not require an initial investment, but hedgers give up any price movement that may have had positive results in the event that the position was left unhedged. Option contracts on the other hand function as insurance for the underlying by providing the downside protection that the hedger seeks and allowing for price movement in the direction that could yield positive results. This insurance does not come without a cost, as we described earlier, since hedgers are required to pay a premium to purchase options.

Example: Hedging with a forward contract

Suppose that a company based in the United States will receive a payment of €10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. **Compute** the following:

1. The value of the €10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 \$/€.

2. The value of the €10M in U.S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.2 \$/€.

Answer:

1. The value at maturity for the hedged position is:
   €10,000,000 × 1.25 \$/€ = \$12,500,000

2. The value at maturity for the unhedged position is:
   €10,000,000 × 1.2 \$/€ = \$12,000,000

Example: Hedging with a put option

Suppose that an investor owns one share of ABC stock currently priced at $30. The investor is worried about the possibility of a drop in share price over the next three months and is contemplating purchasing put options to hedge this risk. **Compute** the following:

1. The profit on the unhedged position if the stock price in three months is $25.
2. The profit on the unhedged position if the stock price in three months is $35.
3. The profit for a hedged stock position if the stock price in three months is $25, the strike price on the put is $30, and the put premium is $1.50.
4. The profit for a hedged stock position if the stock price in three months is $35, the strike price on the put is $30, and the put premium is $1.50.

Answer:

1. Profit $= S_T - S_0 = \$25 - \$30 = -\$5$
2. Profit $= S_T - S_0 = \$35 - \$30 = \$5$
3. Profit $= S_T - S_0 + \max(0, X - S_T) - P_0$
   $= \$25 - \$30 + \max(0, \$30 - \$25) - \$1.50 = -\$1.50$
4. Profit $= S_T - S_0 + \max(0, X - S_T) - P_0$
   $= \$35 - \$30 + \max(0, \$30 - \$35) - \$1.50 = \$3.50$

Professor's Note: Notice that the max term is $5 in Case #3 and $0 in Case #4.

Speculative Strategies

LO 33.6: Calculate and compare the payoffs from speculative strategies involving futures and options.

Speculators have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant **leverage**. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Options also create significant leverage as investors only need to pay the option premium to purchase an option instead of the face value of the underlying. Options differ from futures in that options have asymmetrical payoffs. Gains can be quite large going long options, but losses from long option positions are limited to the option premium.

Example: Speculating with futures

An investor believes that the euro will strengthen against the dollar over the next three months and would like to take a position with a value of €250,000. He could purchase euros in the spot market at 0.80 $/€ or purchase two futures contracts at 0.83 $/€ with an initial margin of $10,000. **Compute** the profit from the following:

1. Purchasing euros in the spot market if the spot rate in three months is 0.85 $/€.
2. Purchasing euros in the spot market if the spot rate in three months is 0.75 $/€.
3. Purchasing the futures contract if the spot rate in three months is 0.85 $/€.
4. Purchasing the futures contract if the spot rate in three months is 0.75 $/€.

Answer:

1. Profit = €250,000 × (0.85 $/€ – 0.80 $/€) = $12,500
2. Profit = €250,000 × (0.75 $/€ – 0.80 $/€) = –$12,500
3. Profit = €250,000 × (0.85 $/€ – 0.83 $/€) = $5,000
4. Profit = €250,000 × (0.75 $/€ – 0.83 $/€) = –$20,000

A summary of these four transactions is as follows:

	Purchase Euros in Spot Market	*Purchase Long Forward Position*
Investment	$200,000	$10,000
Profit if spot at maturity = 0.85 $/€	$12,500	$5,000
Profit if spot at maturity = 0.75 $/€	–$12,500	–$20,000

Example: Speculating with options

An investor who has $30,000 to invest believes that the price of stock XYZ will increase over the next three months. The current price of the stock is $30. The investor could directly invest in the stock, or she could purchase 3-month call options with a strike price of $35 for $3. **Compute** the profit from the following:

1. Investing directly in the stock if the price of the stock is $45 in three months.
2. Investing directly in the stock if the price of the stock is $25 in three months.
3. Purchasing call options if the price of the stock is $45 in three months.
4. Purchasing call options if the price of the stock is $25 in three months.

Answer:

1. Number of stocks to purchase = $30,000 / $30 = 1,000
   Profit = 1,000 × ($45 – $30) = $15,000
2. Profit = 1,000 × ($25 – $30) = –$5,000
3. Number of call options to purchase = $30,000 / $3 = 10,000
   Profit = 10,000 × [max (0, $45 – $35) – $3] = $70,000
4. Profit = 10,000 × [max (0, $25 – $35) – $3] = –$30,000

Professor's Note: Since option contracts are traded in amounts of 100 options, the transactions in #3 and #4 above would entail the purchase of 100 call option contracts (i.e., 10,000 / 100 = 100).

A summary of these four transactions is as follows:

	Purchase Stock	*Purchase Call Option*
# Shares/Call option	1,000	10,000
Profit if stock at maturity = $45	$15,000	$70,000
Profit if spot at maturity = $25	–$5,000	–$30,000

Arbitrage Opportunities

LO 33.7: Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary.

Arbitrageurs are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

Example: Arbitrage of stock trading on two exchanges

Assume stock DEF trades on the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE). The stock currently trades on the NYSE for $32 and on the TSE for ¥2,880. Given the current exchange rate is 0.0105 $/¥, **determine** if an arbitrage profit is possible.

Answer:

Value in dollars of DEF on TSE = ¥2,880 × 0.0105 $/¥ = $30.24

Arbitrageur could purchase DEF on TSE for $30.24 and sell on NYSE for $32.

Profit per share = $32 – $30.24 = $1.76

Risk From Derivatives

LO 33.8: Describe some of the risks that can arise from the use of derivatives.

Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the "bet" one makes starts going in the wrong direction, the results can be catastrophic. Additionally, the risk exists that a trader with instructions to hedge a position may actually use derivatives to speculate. This risk is known as operational risk. Controls need to be carefully established and monitored within both financial and nonfinancial corporations to prevent misuse of derivatives. Risk limits should be set, and adherence to risk limits should be monitored.

Common Terms Related to Derivatives

The following section discusses common terms associated with derivatives. Many of these terms have been mentioned earlier. Understanding these concepts will be helpful going forward as you progress through the derivatives material.

A **derivative** security is a financial security (e.g., options) whose value is derived in part from another security's characteristics or value. This other security is referred to as the underlying asset. A derivative effectively "derives" its price from some other variable.

A **market maker** is the individual that "makes a market" in a security. The market maker maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security, at publicly quoted prices.

A **spot contract** is an agreement to buy/sell an asset *today*. A **forward contract** specifies the price/quantity of an asset to be delivered on or before a future pre-specified date. A **futures contract** is a legally binding agreement to buy/sell a commodity or financial instrument in a designated future month at a previously agreed upon price by the buyer/seller.

A **call option** gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date. A **put option** gives the investor the right to sell a fixed number of shares at a fixed price within a given pre-specified time period. An investor may wish to have the option to sell shares of a stock at a certain price and time in order to hedge an existing investment.

An American-styled option contract can be exercised any time between issue date and expiration date. In contrast, a European-styled option contract may be exercised only on the actual expiration date. **American options** will be worth more than **European options** when the right to early exercise is valuable, and they will have equal value when it is not.

A **long position** refers to actually owning the security, while a short position is when a person sells a security he does not own. An investor taking a short position anticipates a drop in price of the security.

The exercise, or **strike price**, is the price at which the security underlying an options contract may be bought/sold.

Expiration date is the last date on which an option may be exercised.

The **bid price** is the "quoted bid," or the highest price, which a dealer is willing to pay to purchase a security. This is essentially the available price at which an investor can sell shares of stock. The **offer price** is the price at which the security is offered for sale, also known as the "asking price." The **bid-ask spread** is the difference between the ask (a.k.a. offer) price and the bid price.

Hedgers reduce their risks typically through the use of forward contracts or options. By using forward contracts, the trader is attempting to neutralize risk by fixing the price the hedger will pay/receive for the underlying asset. Option contracts, in contrast, are more of an insurance policy.

Speculators want to take a position in the market and profit from this position. Speculators are effectively betting on future price movement. When a speculator uses futures, there is a large possible gain/loss. Speculating using options is less risky since the maximum loss is the cost of the option itself.

Arbitrageurs take offsetting positions in financial instruments in order to lock in a riskless profit.

Key Concepts

LO 33.1

The open outcry system is the more traditional trading system; traders actually indicate their trades through hand signals. Electronic trading involves matching up buyers and sellers electronically.

LO 33.2

The over-the-counter (OTC) market is used for large trades, and a typical OTC trade is conducted over the phone. Terms are not set by an "exchange," giving traders more flexibility to negotiate mutually agreeable terms. The OTC market has more credit risk. Exchanges are organized to eliminate credit risk.

LO 33.3

A call option gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date, while a put option is the right to sell a fixed number of shares at a fixed price within a given pre-specified time period.

A forward contract is an agreement to buy or sell an asset at a pre-selected future time for a certain price.

A futures contract is a more formalized, legally binding agreement to buy or sell a commodity or financial asset in a pre-designated month in the future, at a price agreed upon today by the buyer/seller.

LO 33.4

The payoff on a call option to the option buyer is calculated as follows:

$$\text{Call}_T = \max(0, S_T - X)$$

where:
S_T = stock price at maturity
X = strike price of option

The payoff on a put option is calculated as follows:

$$\text{Put}_T = \max(0, X - S_T)$$

where:
S_T = stock price at maturity
X = strike price of option

The payoff to a long position in a forward contract is calculated as follows:

payoff = $S_T - K$

where:
S_T = spot price at maturity
K = delivery price

LO 33.5

Hedgers use derivatives to control or eliminate a financial exposure. Futures lock in the price of the underlying security and do not allow for any upside potential. Options hedge negative price movements and allow for upside potential since they have asymmetric payouts.

LO 33.6

Speculators use derivatives to make bets on the market. Futures require a small initial investment, which is the initial margin requirement. Futures contracts can result in large gains or large losses as futures have a symmetrical payout function.

LO 33.7

Arbitrageurs seek to earn a riskless profit through the discovery and manipulation of mispriced securities. Riskless profit is earned by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities do not last long as the act of arbitrage brings prices back into equilibrium quickly.

LO 33.8

Derivatives are versatile instruments and can be used for hedging, arbitrage, and pure speculation. Controls need to be carefully established to prevent misuse of derivatives. Risk limits must be carefully established and scrupulously enforced.

Concept Checkers

1. Which of the following statements is an advantage of an exchange trading system? On an exchange system:
   A. terms are not specified.
   B. trades are made in such a way as to reduce credit risk.
   C. participants have flexibility to negotiate.
   D. in the event of a misunderstanding, calls are recorded between parties.

2. Which of the following statements regarding futures contracts is most likely correct? A business with a long exposure to an asset would hedge this exposure by either entering into a:
   A. long futures contract or by buying a call option.
   B. long futures contract or by buying a put option.
   C. short futures contract or by buying a call option.
   D. short futures contract or by buying a put option.

3. Which of the following statements is least likely correct regarding the use of derivatives?
   A. Misuse of derivatives is not a very significant risk.
   B. Risk limits for derivatives should be set, and adherence to these limits should be monitored.
   C. Due to leverage inherent in derivatives, if a bet goes wrong, results can be catastrophic.
   D. There is a risk that traders may use derivatives for unintended purposes.

4. An individual that maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security is a(n):
   A. hedger.
   B. arbitrageur.
   C. speculator.
   D. market maker.

5. An agreement sold over an exchange to buy/sell a commodity or financial instrument at a designated future date is known as a(n):
   A. spot contract.
   B. option contract.
   C. futures contract.
   D. forward contract.

Concept Checker Answers

1. B Exchanges are organized to reduce credit risk. The other answer choices are advantages of over-the-counter trading.

2. D A business with a long exposure to an asset would hedge the exposure by either entering into a short futures contract or by buying a put option.

3. A Misuse of derivatives can be a significant risk for firms that engage in derivatives trading.

4. D A market maker maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security.

5. C A futures contract is an agreement sold on an exchange to buy/sell a commodity or financial instrument in a designated future month.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Mechanics of Futures Markets

Topic 34

Exam Focus

In this topic, candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and convergence of spot prices to futures prices are also likely exam topics. Learn the ways a futures position can be terminated prior to contract expiration and understand how cash settlement is accomplished by the final mark to market at contract expiration.

LO 34.1: Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery and limits.

LO 34.9: Compare and contrast forward and futures contracts.

Futures contracts are exchange-traded obligations to buy or sell a certain amount of an underlying good at a specified price and date. The underlying asset varies from agricultural products to stock indices. Most futures positions are not held to take delivery of the underlying good. Instead, they are closed out or reversed prior to the settlement date.

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price. Futures contracts are used by **speculators** to gain exposure to changes in the price of the asset underlying a futures contract. A **hedger**, in contrast, will use futures contracts to reduce exposure to price changes in the asset (i.e., hedge their asset price risk). An example is a wheat farmer who sells wheat futures to reduce the uncertainty about the price of wheat at harvest time.

Open interest is the total number of long positions in a given futures contract. It also equals the total number of short positions in a futures contract. An open interest of 200 would imply that there are 200 short positions in existence and 200 long positions in existence. It is possible, on any given day, for the trading volume on a contract to be higher than its open interest.

Trading Futures Contracts

To illustrate how a futures contract is created, let's use a contract on gold as an example. Each contract represents 100 troy ounces and is quoted on a per-ounce basis. Suppose an investor instructs a broker to sell one futures contract on gold with an April delivery date. At about the same time another investor instructs a broker to buy an identical futures

contract. The seller of the futures contract has a short-futures position and is obligated to sell 100 ounces of gold at the futures price at contract expiration. The buyer of the futures contract has a long futures position and is obligated to buy 100 ounces of gold at the futures price at maturity. They agree on a price of $993.60 per ounce. The two parties in this example have no idea of one another's existence because the clearinghouse (discussed in LO 34.4) takes the opposite side of every transaction. In the futures market there is always the same number of long and short positions. This means that if a long position wins, the corresponding short position loses.

Characteristics Specified in a Futures Contract

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized. When a new futures contract is introduced to the marketplace, the futures exchange must specify the exact terms of the contract. Futures contract characteristics specified by the exchange include the following:

- *Quality of the underlying asset.* When the underlying asset for the contract is a financial asset, such as Japanese yen, the definition of the asset is straightforward. However, when the underlying asset is a commodity, there may be different levels of quality for that good available in the marketplace (e.g., different types of wheat). The futures exchange stipulates the quality of a good that will be acceptable for settling the contract.
- *Contract size.* The contract size specifies the quantity of the asset that must be delivered to settle a futures contract (e.g., one grain contract = 5,000 bushels).
- *Delivery location.* The exchange specifies the place where delivery will take place.
- *Delivery time.* Futures contracts are referred to by the month in which delivery is to take place (e.g., a December corn contract). Some contracts are not settled by delivery but by payment in cash, based on the difference between the futures price and the market price at settlement.
- *Price quotations and tick size.* The exchange determines how the price of a contract will be quoted as well as the minimum price fluctuation for the contract, which is referred to as the **tick size**. For example, grain is quoted in dollars per bushel, and the minimum tick size is ¼ cent per bushel. Since a grain contract consists of 5,000 bushels, the minimum tick size is $12.50 (= 5,000 × $0.0025) per contract.
- *Daily price limits.* The exchange sets the maximum price movement for a contract during a day. For example, wheat cannot move more than $0.20 from its close the preceding day, for a daily price limit of $1,000. When a contract moves down by its daily price limit, it is said to be **limit down**. When the contract moves up by its price limit, it is said to be **limit up**.
- *Position limits.* The exchange sets a maximum number of contracts that a speculator may hold in order to prevent speculators from having an undue influence on the market. Such limits do not apply to hedgers.

Futures/Spot Convergence

LO 34.2: Explain the convergence of futures and spot prices.

The spot (cash) price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The **basis** is the difference between the spot price and the futures price.

basis = spot price – futures price

As the maturity date nears, the basis converges toward zero. At expiration, the spot price must equal the futures price because the futures price has become the price today for delivery today, which is the same as the spot. Arbitrage will force the prices to be the same at contract expiration.

Example: Why the futures price must equal the spot price at expiration

Suppose the current spot price of silver is $4.65. **Demonstrate** by arbitrage that the futures price of a futures silver contract that expires in one minute must equal the spot price.

Answer:

Suppose the futures price was $4.70. We could buy the silver at the spot price of $4.65, sell the futures contract, and deliver the silver under the contract at $4.70. Our profit would be $4.70 – $4.65 = $0.05. Because the contract matures in one minute, there is virtually no risk to this arbitrage trade.

Suppose instead the futures price was $4.61. Now we would buy the silver contract, take delivery of the silver by paying $4.61, and then sell the silver at the spot price of $4.65. Our profit is $4.65 – $4.61 = $0.04. Once again, this is a riskless arbitrage trade.

Therefore, in order to prevent arbitrage, the futures price at the maturity of the contract must be equal to the spot price of $4.65.

Operation of Margins

LO 34.3: Describe the rationale for margin requirements and explain how they work.

Margin is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the **initial margin**. The **maintenance margin** is the minimum margin

account balance required to retain the futures position. When the margin account balance falls below the maintenance margin, the investor gets a margin call, and he must bring the margin account back to the initial margin amount. The amount necessary to do this is called the **variation margin**.

Example: Margin trading

Let's return to our investor with the long gold contract. The investor entered the position at \$993.60. Each contract controls 100 troy ounces for a current market value of \$99,360. Assume that the initial margin is \$2,500, the maintenance margin is \$2,000, and the futures price drops to \$991.00 at the end of the first day and \$985.00 on the end of the second day. **Compute** the amount in the margin account at the end of each day for the long position and any variation margin needed.

Answer:

At the end of the first day, the loss is computed as (\$991 – \$993.6)100 = –\$260, so when the account is marked to market, \$260 is withdrawn from the buyer's margin account and \$260 deposited in the seller's margin account. The buyer's (long) margin account balance is now \$2,240 (= \$2,500 – \$260). The margin account balance for the short position is now \$2,760 (= \$2,500 + \$260).

At the end of the second day, the daily loss is (\$985 – \$991)100 = –\$600, and the buyer's margin account balance is reduced to \$1,640 (= \$2,240 – \$600). At \$1,640 the investor will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is the amount necessary to bring the margin account back up to the initial margin. In this case, it is \$860 (= \$2,500 – \$1,640).

Depending on the client, brokers may require the posting of a balance in the margin account more than the maintenance margin requirements established by exchanges. For example, hedgers are usually required to post smaller margins than speculators. To ensure that the daily cash flows are withdrawn or contributed appropriately, the exchange has a clearinghouse.

Clearinghouses in Futures Transactions

LO 34.4: Describe the role of a clearinghouse in futures and over-the-counter market transactions.

Each exchange has a **clearinghouse**. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer. By doing this, the clearinghouse allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the

counterparty defaulting since the counterparty is now the clearinghouse. In the history of U.S. futures trading, the clearinghouse has never defaulted on a trade.

The clearinghouse has members that collateralize it, ensuring that no defaults take place. All trades eventually go through the clearinghouse members, who must have a **clearing margin** posted at the clearinghouse in the same way an investor has a margin account with a broker. This ensures that the clearinghouse is liquid enough at all times to honor all obligations under futures contracts.

Over-the-Counter Markets

LO 34.5: Describe the role of collateralization in the over-the-counter market and compare it to the margining system.

The over-the-counter (OTC) market includes the trading in all securities not listed on one of the registered exchanges. This market is subject to a good deal of credit risk since the party on the other side of an OTC contract could default on its payments. One way to reduce this credit risk is by means of **collateralization**. Collateralization is basically a marked to market feature for the OTC market where any loss is settled in cash at the end of the trading day. A cash payment is made to the party with a positive account balance. This is a similar system to trading on margin where the futures trader needs to restore funds if the value of the contract drops below the maintenance margin.

Recently passed legislation requires that some OTC transactions use clearinghouses. OTC market clearinghouses operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the clearinghouse for acceptance. Assuming the transaction is accepted, the clearinghouse will become the counterparty to both parties X and Y. Thus, the clearinghouse assumes the credit risk of both parties in an OTC transaction. This risk is managed by requiring the parties to post initial margin and any variation margins on a daily basis.

Historically, OTC markets have functioned as a series of bilateral agreements between parties. If a clearinghouse was instead used for every OTC transaction, each market participant would only deal with a central clearing party. However, because only some OTC transactions are currently required to use clearinghouses, in practice, the current OTC market is a mix of both bilateral agreements and transactions dealing with one or more clearinghouses.

Arguments for the use of clearinghouses in OTC markets include: (1) automatic posting of collateral, (2) reduction of financial system credit risk, and (3) increased transparency of OTC trades. Governments have pushed for the use of clearinghouses in OTC markets in an attempt to reduce systemic risk, which is the risk that a failure by a significant financial institution will impact other institutions and potentially lead to a collapse of the overall financial system. An example of systemic risk occurred during the 2007–2009 credit crisis when the OTC transactions for insurance corporation AIG led to large losses and an eventual bailout of the company by the U.S. government.

Normal and Inverted Futures Market

LO 34.6: Identify the differences between a normal and inverted futures market.

The **settlement price** is analogous to the closing price for a stock but is not simply the price of the last trade. It is an average of the prices of the trades during the last period of trading, called the closing period, which is set by the exchange. This feature of the settlement price prevents manipulation by traders. The settlement price is used to make margin calculations at the end of each trading day.

Depending on the direction of futures settlement prices, the market may be normal or inverted. Increasing settlement prices over time indicates a **normal market**. Conversely, decreasing settlement prices over time indicates an **inverted market**.

The Delivery Process

LO 34.7: Describe the mechanics of the delivery process and contrast it with cash settlement.

There are four ways to terminate a futures contract:

1. A short can terminate the contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is called **delivery**. The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the **notice of intention to deliver** file. Each exchange has specific rules as to the conditions for making an intent to deliver. However, the price paid or received will be dictated by the settlement period on the exchange-determined last trading day of the contract.

2. In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.

3. You may make a **reverse**, or **offsetting**, trade in the futures market. With futures, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled. The contract price can differ between the two contracts. If you initially are long one contract at $970 per ounce of gold and subsequently sell (i.e., take the short position in) an identical gold contract when the price is $950 per ounce, $20 multiplied by the number of ounces of gold specified in the contract will be deducted from the margin deposit(s) in your account. The sale of the futures contract ends the exposure to future price fluctuations on the first contract. Your position has been *reversed*, or **closed out**, by a *closing* trade.

4. A position may also be settled through an **exchange for physicals**. Here you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange (i.e., an ex-pit transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened.

An exchange for physicals differs from a delivery in that the traders actually exchange the goods, the contract is not closed on the floor of the exchange, and the two traders privately negotiate the terms of the transaction. Regular delivery involves only one trader and the clearinghouse.

Types of Orders

LO 34.8: Evaluate the impact of different trading order types.

There are several different types of orders in the marketplace:

Market orders are orders to buy or sell at the best price available. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

Limit orders are orders to buy or sell away from the current market price. A *limit buy order* is placed below the current price. A *limit sell order* is placed above the current price. Limit orders have a time limit, such as instantaneous, one day, one week, one month, or good till canceled. Limit orders are turned over to the specialist by the commission broker.

Stop-loss orders are used to prevent losses or to protect profits. Suppose you own a stock currently selling for $40. You are afraid that it may drop in price, and if it does, you want your broker to sell it, thereby limiting your losses. You would place a *stop loss sell* order at a specific price (e.g., $35); if the stock price drops to this level, your broker will place a sell market order. A *stop loss buy* order is usually combined with a short sale to limit losses. If the stock price rises to the "stop" price, the broker enters a market order to buy the stock.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. The stop price and limit price must be specified, so that once the stop level is reached, or bettered, the order would turn into a limit order and hopefully transact at the limit price. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time period for the order (**time-of-day order**). **Good-till-canceled** (GTC) **orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

Regulatory, Accounting, and Tax Frameworks

Regulation

In the United States, the **Commodity Futures Trading Commission** (CFTC) is responsible for regulating futures markets. The CFTC licenses futures exchanges as well as traders who offer futures trading services to the public. It also approves new futures contracts and any revisions to existing futures contracts. When approving contracts, the agency ensures that each contract serves a useful economic purpose (e.g., for either hedging or speculating).

In addition, the CFTC is responsible for communicating prices to the public, addressing public complaints, and taking disciplinary actions against members who violate futures exchange rules.

Other regulatory bodies that influence the futures markets include the National Futures Association (NFA), the Securities and Exchange Commission (SEC), the Federal Reserve Board, and the U.S. Treasury Department. The SEC, Fed, and Treasury Department are mainly concerned with how futures trading impacts spot market trading in stocks and bonds. The NFA has a more prominent role by attempting to prevent fraud and ensuring that futures markets operate in the best interests of the public. Examples of futures trading fraud include cornering the market (i.e., taking excessive long positions while influencing the supply of the commodity underlying the long futures contracts) and front running (traders using privileged information to trade in their own accounts before customer accounts).

Accounting

When accounting for changes in the market value of a futures contract, changes must be recognized when they occur. The exception to this accounting standard is when a futures contract is being used for hedging purposes. **Hedge accounting** specifies that gains/losses from a hedging instrument be recognized in the same period as gains/losses from the asset being hedged.

Under FAS 133 [Financial Accounting Standard Board (FASB) Statement No. 133], the fair market value of all derivative contracts must be included on the balance sheet. In addition to more position transparency, FAS 133 places stricter guidelines on the use of hedge accounting. To use this accounting method, it must be shown that the hedging instrument frequently and effectively offsets the intended risk exposure.

Taxes

Regarding U.S. tax regulations, differences arise due to the nature of taxable gains/losses and the timing of realized gains/losses. For corporate taxpayers, capital gains are taxed at the same level as ordinary income and capital losses are restricted. For non-corporate taxpayers, capital gains are taxed at the same level as ordinary income, but long-term gains (investments held over one year) are subject to a maximum 15% tax rate. Another difference is that capital losses are deductible for non-corporate taxpayers.

For tax purposes, futures contracts are considered closed out at the end of each year. This gives rise to a 60/40 rule for non-corporate taxpayers where capital gains/losses are treated as 60% long term and 40% short term. This rule, however, does not apply to hedging activities. Using futures for hedging purposes must be declared on the same day the transaction is entered. Gains/losses on hedging transactions are taxed at the same rate as ordinary income.

Key Concepts

LO 34.1

A long (short) futures position obligates the owner to buy (sell) the underlying asset at a specified price and date. Most futures positions are reversed (or closed out) as opposed to satisfying the contract by making (or taking) delivery.

LO 34.2

The spot price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The basis is the difference between the spot price and the futures price. As the maturity date nears, the basis converges toward zero. Arbitrage will force the spot and futures prices to be the same at contract expiration.

LO 34.3

Futures are traded on margin (leveraged):

- Initial margin is the necessary collateral to trade the futures.
- Maintenance margin is the minimum collateral amount required to retain trading privileges.
- Variation margin is the collateral amount that must be deposited to replenish the margin account back to the initial margin.

The futures market is a zero-sum game in that the short's losses are the long's gains and vice versa. Gains and losses due to changes in futures prices are computed at the end of each trading day in a process known as marking to market.

LO 34.4

The clearinghouse maintains an orderly and liquid market by acting as the counterparty to each long or short futures position. In the over-the-counter (OTC) markets, the clearinghouse also becomes the counterparty to both parties in an OTC transaction.

LO 34.5

Collateralization is a means of reducing credit risk in OTC contracts.

LO 34.6

The futures settlement price is an average of the prices of the trades during the last period of trading, called the closing period. It is used to make margin calculations at the end of each trading day. Increasing settlement prices over time indicate a normal market, while decreasing settlement prices over time indicate an inverted market.

LO 34.7

A short can terminate the futures contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is known as the delivery process. In a cash-settlement contract, delivery is not an option.

LO 34.8

Several different types of orders exist in the marketplace including: market, limit, stop-loss, stop-limit, and market-if-touched orders. Market orders are orders to buy or sell at the best price available. Limit orders are orders to buy or sell away from the current market price. Stop-loss orders are used to prevent losses or to protect profits. Stop-limit orders are a combination of a stop and limit order. Market-if-touched orders are orders that would become market orders once a specified price is reached.

LO 34.9

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized.

Concept Checkers

1. When an investor is obligated to buy the underlying asset in a futures position, it is a:
   A. basis trade.
   B. long-futures position.
   C. short-futures position.
   D. hedged-futures position.

2. Which of the following are characteristics specified by a futures contract?
   I. Asset quality and asset quantity.
   II. Delivery arrangements and delivery time.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. An investor enters into a short position in a gold futures contract with the following characteristics:
   - The initial margin is $3,000.
   - The maintenance margin is $2,250.
   - The contract price is $1,300.
   - Each contract controls 100 troy ounces.

   If the price drops to $1,295 at the end of the first day and $1,290 at the end of the second day, which of the following is closest to the variation margin required at the end of the second day?
   A. $0.
   B. $250.
   C. $500.
   D. $1,000.

4. Which of the following items are functions of the clearinghouse?
   I. Determine which contracts trade.
   II. Receive margin deposits from brokers.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. It is possible that which of the following types of orders may never be executed?
   A. Limit orders.
   B. Market-if-touched (MIT) orders.
   C. Stop-limit orders.
   D. All of the above.

Concept Checker Answers

1. **B** When an investor is obligated to buy the underlying asset in a futures position, it is a long futures position.

2. **C** Delivery time, asset quality, asset quantity, and delivery arrangements are all characteristics specified by the futures contract.

3. **A** Note that the investor in this question has a short position that profits from price declines. The short position margin account has increased by $1,000 over the two days, so there is no variation margin required.

4. **B** The clearinghouse acts as buyer to every seller and seller to every buyer, thus virtually eliminating default risk. It also collects margin payments from clearing members (brokers). Determining which contracts will trade is a function of the exchange, not the clearinghouse.

5. **D** All of these orders require that the price reach a certain range before being activated. If the price never reaches that range, the order will never be activated.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Hedging Strategies Using Futures

Topic 35

Exam Focus

Futures contracts are used extensively for implementing hedging strategies. This topic presents the calculations for determining the optimal hedge ratio and shows how to use it to determine the number of futures contracts necessary to hedge a spot market exposure. This topic also addresses basis risk, the change in the relationship between spot prices and futures prices over a hedge horizon. Basis risk arises because an asset being hedged may not be exactly the same as the asset underlying the futures contract.

Hedging With Futures

LO 35.1: Define and differentiate between short and long hedges and identify their appropriate uses.

A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

Advantages and Disadvantages of Hedging

LO 35.2: Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.

The objective of hedging with futures contracts is to reduce or eliminate the price risk of an asset or a portfolio. For example, a farmer with a large corn crop that will be harvested in a few months could wait until the end of the growing season and sell his corn at the prevailing spot price, *or* he could sell corn futures and "lock in" the price of his corn at a predetermined rate. By taking a short position in a corn futures contract, the farmer eliminates—or at least reduces—exposure to fluctuating corn prices. This is an example of a *short hedge*, where the user locks in a future selling price.

Alternatively, a cereal company will need to purchase corn in the future. The company could wait to buy corn in the spot market and face the volatility of future corn spot prices or lock in its purchase price by buying corn futures in advance. This demonstrates an *anticipatory hedge*. The cereal company has an anticipated need for corn and buys corn futures to lock in the price of those future corn purchases. This is an example of a *long hedge*, where the user locks in a future purchasing price.

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability. However, there are some arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge.

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates. If competitors do not hedge, then there is an incentive to keep the status quo. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes.

Basis Risk

LO 35.3: Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures.

LO 35.4: Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness.

When all of the existing position characteristics match perfectly with those of the futures contract specifications, we have a perfect hedge. With a perfect hedge, the loss on a hedged position will be perfectly offset by the gain on the futures position. Perfect hedges are not very common. There are two major reasons why this is so: (1) the asset in the existing position is often not the same as that underlying the futures (e.g., we may be hedging a corporate bond portfolio with a futures contract on a U.S. Treasury bond), and (2) the hedging horizon may not match perfectly with the maturity of the futures contract. The existence of either one of these conditions leads to what is called **basis risk**.

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). Basis is calculated as:

basis = spot price of asset being hedged – futures price of contract used in hedge

When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.

Professor's Note: This is the typical definition for basis (where basis equals spot price minus futures price). However, basis is also sometimes defined as: futures price minus spot price, mostly when dealing with financial asset futures.

When the spot price increases faster than the futures price over the hedging horizon, basis increases and a strengthening of the basis is said to occur. When the futures price increases faster than the spot price and the basis decreases, a weakening of the basis occurs. When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is termed *basis risk*, and it can work either for or against a hedger.

To minimize basis risk, hedgers should select the contract on an asset that is most highly correlated with the spot position and a contract maturity that is closest to the hedging horizon. Contract liquidity must also be considered when selecting a futures contract for hedging.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset. Let's discuss each of these sources in more detail.

1. *Interruption in the convergence of the futures and spot prices.* Normally, spot prices and futures prices will converge as the time to maturity decreases, and basis reduces to zero at maturity. However, if the position is unwound prior to maturity, the return to the futures position could be different from the return to the cash position. A more rapid convergence results in a more rapid transfer of margin payments, while a less rapid convergence would delay payments. An interruption in the convergence could result in payments from the seller to the buyer. All of these effects are types of basis risk.

2. *Changes in the cost of carry.* Significant basis risk can arise due to changes in the components of the cost of carry. The cost of carry includes storage and safekeeping, interest, insurance, and related costs. Perhaps the most volatile of these costs is interest costs. An increase in the interest rates increases the opportunity cost of holding the asset, so the cost of carry and, hence, the basis of the contract rises.

3. *Imperfect matching between the cash asset and the hedge asset.* Sometimes it may be more efficient to **cross hedge** or hedge a cash position with a hedge asset that is closely related but different from the cash asset. For example, Eurodollar deposits are closely related to T-bill rates and may be considered a good hedge. However, if there is a structural shock that changes the close relationship of these two assets, the position may not be hedged as effectively as originally believed. This is the most common form of basis risk. Other forms of mismatch include maturity or duration mismatches, liquidity mismatches, and credit risk mismatches:
   - *Maturity or duration mismatch.* Hedging a portfolio of mortgages with 10-year Treasury notes (T-notes) may seem reasonable if the effective duration of the mortgages matches the duration of the T-notes. However, if rates fall and the mortgages prepay faster (resulting in a shorter duration), the position will not be matched.
   - *Liquidity mismatch.* Hedging an illiquid asset with a more liquid one will result in greater basis risk. Although over the long term the prices may be comparable, the difference in liquidity may result in large gaps between the pricing of the two assets. Hence, basis risk is inversely proportional to the liquidity of the hedged asset.

- *Credit risk mismatch.* The widening or narrowing of credit spreads constitutes another form of basis risk when the credit risk of the hedged asset is different (or becomes different) from the credit risk of the hedge instrument.

All of these represent basis risk. The size and type of basis risk can vary during the term of the contract, even if the position is perfectly hedged at maturity.

The Optimal Hedge Ratio

We can account for an imperfect relationship between the spot and futures positions by calculating an **optimal hedge ratio** that incorporates the degree of correlation between the rates.

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The *optimal hedge ratio*, which minimizes the variance of the combined hedge position, is defined as follows:

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

This is also the beta of spot prices with respect to futures contract prices since:

$$\rho = \frac{Cov_{S,F}}{\sigma_S\sigma_F} \text{ and } \frac{Cov_{S,F}}{\sigma_S\sigma_F} \times \frac{\sigma_S}{\sigma_F} = \frac{Cov_{S,F}}{\sigma_F{}^2} = \beta_{S,F}$$

where:
$\rho_{S,F}$ = the correlation between the spot prices and the futures prices
σ_S = the standard deviation of the spot price
σ_F = the standard deviation of the futures price

Example: Minimum variance hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be \$0.10, and the annual standard deviation of the futures price to be \$0.125. **Compute** the hedge ratio.

Answer:

$$HR = 0.925 \times \frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

The **effectiveness of the hedge** measures the variance that is reduced by implementing the optimal hedge. This effectiveness can be evaluated with a coefficient of determination (R^2) term where the independent variable is the change in futures prices and the dependent variable is the change in spot prices. Recall that R^2 measures the goodness-of-fit of a regression. As shown previously, the beta of spot prices with respect to futures prices is equal

to the hedge ratio (HR), which is also the slope of this regression. The R^2 measure for this simple linear regression is the square of the correlation coefficient (ρ^2) between spot and futures prices.

Hedging With Stock Index Futures

LO 35.5: Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the "tailing the hedge" adjustment.

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

Example: Hedging with stock index futures

You are a portfolio manager with a \$20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 1,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. **Identify** whether a long or short hedge is appropriate, and **determine** the number of S&P 500 contracts you need to implement the hedge.

Answer:

You are long the S&P 500, so you should construct a short hedge and sell the futures contract. The number of contracts to sell is equal to:

$$1.4 \times \frac{\$20{,}000{,}000}{1{,}150 \times 250} \approx 97 \text{ contracts}$$

Tailing the Hedge

A hedger may actually over-hedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of over-hedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot price to futures price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

Example: Tailing the hedge

Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period you find that the S&P 500 spot price is 1,095 and the futures price is now 1,160. **Determine** the number of S&P 500 contracts needed after making a tailing the hedge adjustment.

Answer:

The number of contracts to sell is equal to:

1.4 × [($20,000,000) / (1,150 × 250)] × (1,095 / 1,160) = 92 contracts

Adjusting the Portfolio Beta

LO 35.6: Explain how to use stock index futures contracts to change a stock portfolio's beta.

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let β be our portfolio beta, β^* be our target beta after we implement the strategy with index futures, P be our portfolio value, and A be the value of the underlying asset (i.e., the stock index futures contract). To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$$

This equation can result in either positive or negative values. Negative values indicate selling futures (decreasing systematic risk), and positive values indicate buying futures contracts (increasing systematic risk).

Example: Adjusting portfolio beta

Suppose we have a well-diversified $100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the 3-month S&P 500 Index is 1,080. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. **Demonstrate** how to adjust the portfolio's beta.

Answer:

In this instance, our target beta, β^*, is 0, since a complete hedge is desired.

$$\text{number of contracts} = (0 - 1.2)\frac{100{,}000{,}000}{1{,}080 \times 250} = -444.44$$

The negative sign tells us we need to sell 444 contracts.

Rolling a Hedge Forward

LO 35.7: Explain the term "rolling the hedge forward" and describe some of the risks that arise from this strategy.

When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward**.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is referred to as rollover basis risk, or simply **rollover risk**.

Key Concepts

LO 35.1

Hedging may be achieved by shorting futures to protect an underlying position against price deterioration or by buying futures to hedge against unanticipated price increases in an underlying asset.

LO 35.2

Investors hedge with futures contracts to reduce or eliminate the price risk of an asset or a portfolio. The advantage of hedging is that it leads to less uncertainty regarding future profitability. The disadvantage of hedging is that it can lead to less profitability if the asset being hedged ends up increasing in value.

LO 35.3

Basis risk is the risk that a difference may occur between the spot price of a hedged asset and the futures price of the contract used to implement the hedge. Basis risk is zero only when there is a perfect match between the hedged asset and the contract's underlying instrument in terms of maturity and asset type.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset.

LO 35.4

Sometimes it may be more efficient to cross hedge or hedge a cash position with a hedge asset that is closely related but different from the cash asset.

A hedge ratio is the ratio of the size of the futures position relative to the spot position necessary to provide a desired level of protection.

$$\text{HR} = \rho_{\text{spot,futures}} \times \frac{\sigma_{\text{spot}}}{\sigma_{\text{futures}}}$$

The effectiveness of the hedge measures the variance that is reduced by implementing the optimal hedge.

LO 35.5

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). The number of futures contracts required to completely hedge an equity position is determined as follows:

$$\#\text{ of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)$$

LO 35.6

When hedging an equity portfolio with a short position in stock index futures, the beta of the portfolio is reduced. To change a stock portfolio's beta, use the following formula:

$$\text{number of contracts} = \left(\beta^{*} - \beta\right) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

LO 35.7

When the hedging horizon is longer than the maturity of the futures, the hedge must be rolled forward to retain the hedge. This exposes the hedger to rollover risk, the basis risk when the hedge is re-established.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An equity portfolio is worth $100 million with the benchmark of the Dow Jones Industrial Average. The Dow is currently at 10,000, and the corresponding portfolio beta is 1.2. The futures multiplier for the Dow is 10.

1. Which of the following is the closest to the number of contracts needed to double the portfolio beta?
   A. 1,100.
   B. 1,168.
   C. 1,188.
   D. 1,200.

2. To cut the beta in half, the correct trade is:
   A. long 600 contracts.
   B. short 600 contracts.
   C. long 1,200 contracts.
   D. short 1,200 contracts.

3. Which of the following situations describe a hedger with exposure to basis risk?
   I. A portfolio manager for a large-cap growth fund knows he will be receiving a significant cash investment from a client within the next month and wants to pre-invest the cash using stock index futures.
   II. A farmer has a large crop of corn he is looking to sell before June 30. The farmer uses a June futures contract to lock in his sales price.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. The standard deviation of price changes in a wheat futures contract is 0.6, while the standard deviation of changes in the price of wheat is 0.75. The covariance between the spot price changes and the futures price changes is 0.3825. Which of the following is closest to the optimal hedge ratio?
   A. 0.478.
   B. 0.850.
   C. 1.063.
   D. 1.250.

5. A large-cap value equity manager has a \$6,500,000 equity portfolio with a beta of 0.92. An S&P 500 futures contract is available with a current value of 1,175 and a multiplier of 250. What position should the manager take to completely hedge the portfolio's market risk?
   A. Short 20 contracts.
   B. Short 22 contracts.
   C. Short 24 contracts.
   D. Long 22 contracts.

Concept Checker Answers

1. D $(2.4-1.2)\dfrac{100,000,000}{10,000\times 10}=1.2\times 1,000=1,200$

   where beta = 1.2, target beta = 2.4, A = 10 × 10,000, P = \$100 million

2. B $(0.6-1.2)\dfrac{100,000,000}{10,000\times 10}=-0.6\times 1,000=-600$

   where beta = 1.2, target beta = 0.6, A = 10 × 10,000, P = \$100 million

3. C Both of these situations describe exposure to basis risk—the risk that the difference between the spot price and futures delivery price will change. The portfolio manager using futures to pre-invest the cash does not know the exact date he will receive the cash and may need to sell or hold the futures contract for a longer time period than intended. The farmer may need to sell his June futures contract early if he sells his corn earlier than the June futures expiration date.

4. C Notice in this problem, we were given the covariance but not the correlation. We can calculate the correlation using the formula learned back in the Quantitative Analysis material, as follows:

   $$\rho=\frac{COV_{S,F}}{(\sigma_S)(\sigma_F)}=\frac{0.3825}{(0.75)(0.60)}=0.85$$

   Now that we have our correlation value, we can calculate the minimum hedge ratio as:

   $$0.85\left(\frac{0.75}{0.60}\right)=1.0625,\ \text{or, directly,}\ \frac{Cov_{S,F}}{\sigma_F^{\,2}}=\frac{0.3825}{0.6^2}=1.0625$$

5. A $0.92\times\dfrac{6,500,000}{1,175\times 250}\approx 20$ contracts

   Because the manager has a long position in the market, she will want to take a short position in the futures.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Interest Rates

Topic 36

Exam Focus

Spot, or zero, rates are computed from coupon bonds using a method known as bootstrapping. Forward rates can then be computed from the spot or zero curve. For the exam, understand how to use the bootstrapping method and how to compute forward rates from spot rates. Also, be familiar with the discrete and continuous compounding methods. Note that the fixed income readings in Book 4 will provide more information on the calculation of spot and forward rates as well as constructing the spot and forward rate curves. Duration and convexity are also mentioned in this topic but will be discussed in much more detail in Book 4.

Types of Rates

LO 36.1: Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the "risk-free" rate.

Three interest rates play a key role in interest rate derivatives: Treasury rates, LIBOR, and repo rates. Keep in mind that interest rates increase as the credit risk of the underlying instrument increases.

- **Treasury rates.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** The London Interbank Offered Rate (LIBOR) is the rate at which large international banks fund their activities. Some credit risk exists with LIBOR.
- **Repo rates.** The "repo" or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price. The interest rate implied by the price differential is the repo rate. The most common repo is the overnight repurchase agreement. Longer-term agreements are called term repos. Depending on the parties and structure involved, there is some credit risk with repurchase agreements.

Professor's Note: You may see reference to an inverse floater (a.k.a. reverse floater) on the exam. Just know that an inverse floater is a debt instrument whose coupon payments fluctuate inversely with the reference rate (e.g., LIBOR). For example, the inverse floater's coupon rate will increase when LIBOR decreases and vice versa.

As mentioned, Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates. However, derivative traders view these rates as being too low to be considered risk free (since part of the demand for these bonds comes from fulfilling regulatory requirements, which drives prices up and rates down). As a result, traders instead use LIBOR rates for short-term risk-free rates, because LIBOR better reflects a trader's opportunity cost of capital.

Compounding

LO 36.2: Calculate the value of an investment using different compounding frequencies.

LO 36.3: Convert interest rates based on different compounding frequencies.

Derivative pricing often uses a framework called continuous time mathematics. In this framework, it is assumed that returns are continuously compounded. This is a theoretical construct only, as returns cannot literally be compounded continuously. Fortunately, converting discrete compounding to continuous compounding is straightforward.

If we have an initial investment of *A* that earns an annual rate *R*, compounded *m* times a year for *n* years, then it has a future value of:

$$FV_1 = A\left(1+\frac{R}{m}\right)^{m\times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R\times n}$$

For any rate, *R*, FV_2 will always be greater than FV_1. The difference will decrease as *m* increases. In fact, as *m* becomes infinitely large, the difference goes to zero.

In most circumstances rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. Using the previous two equations, the goal is to solve the following:

$$A\left(1+\frac{R}{m}\right)^{m\times n} = Ae^{R_c n}$$

where:
R_c = the continuous rate

We can solve for R_c as:

$$R_c = m \times \ln\left(1+\frac{R}{m}\right)$$

We can also solve for *R* as:

$$R = m\left(e^{R_c/m} - 1\right)$$

Professor's Note: In order to algebraically solve for R or R_c, given one of the equations above, it is helpful to understand that e is the base of the natural log (ln). In other words, the natural log is the inverse function of the exponential function: $e^{\ln(x)} = \ln(e^x) = x$. So if you are given an equation such that $R = e^x$; x will be equal to: ln(R).

Example: Computing continuous rates

Suppose we have a 5% rate that is compounded semiannually. **Compute** the corresponding continuous rate. Repeat this for quarterly, monthly, weekly, and daily compounding.

Answer:

$$R_c = 2\ln\left(1+\frac{0.05}{2}\right) = 0.049385$$

The results for other compounding frequencies are shown in Figure 1.

Figure 1: Compounding Frequencies and Returns

m	R_c
4	0.049690
12	0.049896
52	0.049976
250	0.049995

Notice that as m increases, the difference between the rates decreases.

Example: Discrete compounding rate

A loan is quoted at 12% annually with continuous compounding. Interest is paid monthly. **Calculate** the equivalent rate with monthly compounding.

Answer:

$$R = 12(e^{0.12/12} - 1) = 12.06\%$$

Spot (Zero) Rates and Bond Pricing

LO 36.4: Calculate the theoretical price of a bond using spot rates.

Spot rates are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called zero rates. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

Bond Pricing

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds, and its value, assuming continuous compounding and semiannual coupons, is:

$$B = \left(\frac{c}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2} \times j} \right) + \left(FV \times e^{-\frac{z_N}{2} \times N} \right)$$

where:
c = the annual coupon
N = the number of semiannual payment periods
z_j = the bond equivalent spot rate that corresponds to j periods (j/2 years) on a continuously compounded basis
FV = the face value of the bond

Don't let this formula intimidate you. It simply says that the value of a bond is the present value of its cash flows, where each cash flow is discounted at the appropriate spot rate for its maturity. Notice that the negative sign on the rate just means that the coupon and principal payments are being discounted back to the present in a continuous fashion. The following example is a good illustration of the process.

Example: Calculating bond price

Compute the price of a $100 face value, 2-year, 4% semiannual coupon bond using the annualized spot rates in Figure 2.

Figure 2: Spot Rates

Maturity (Years)	*Spot Rate (%)*
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

Answer:

$$B = \left(\$2 \times e^{-\frac{0.025}{2} \times 1} \right) + \left(\$2 \times e^{-\frac{0.026}{2} \times 2} \right) + \left(\$2 \times e^{-\frac{0.027}{2} \times 3} \right) + \left(\$102 \times e^{-\frac{0.029}{2} \times 4} \right) = \$102.10$$

Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

Example: Calculating bond yield

Compute the yield for the bond in the previous example.

Answer:

PMT = 2; N = 4; PV = −102.10; FV = 100; CPT → I/Y = 1.456;

Y = 1.456% × 2 ≈ 2.91%

The bond's **par yield** is the rate which makes the price of a bond equal to its par value. When the bond is trading at par, the coupon will be equal to the bond's yield.

Bootstrapping Spot Rates

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125% How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for z_1 in the bond valuation equation, given the price, as follows:

$$\$102.2969 = \left(\$100 + \frac{\$6.125}{2}\right) \times e^{-\frac{z_1}{2}}$$

Solving this for z_1:

$$z_1 = -2 \times \ln\left[\frac{\$102.2969}{\left(\$100 + \frac{\$6.125}{2}\right)}\right] = 1.491\%$$

The 6-month spot rate on a bond equivalent basis is 1.491%. Also note that the yield to maturity did not need to be computed in this case because the yield to maturity (YTM) and the spot rate are the same.

How is the spot rate that corresponds to one year found? Suppose a T-bond that matures in one year is priced at 104.0469% of par and has a semiannual coupon of 6.25%. From the previous computation, the 6-month spot rate is known, so the bond valuation equation can be written as:

$$\$104.0469 = \left(\frac{\$6.25}{2} \times e^{-\frac{0.01491}{2}}\right) + \left(\$100 + \frac{\$6.25}{2}\right) \times e^{-\frac{z_2}{2} \times 2}$$

$$\Rightarrow z_2 = 0.02136 = 2.136\%$$

The 1-year spot rate with continuous compounding is 2.136%.

Example: Bootstrapping spot rates

Compute the corresponding spot rate curve using the information in Figure 3. Note that we've already computed the first two spot rates.

Figure 3: Input Information to Bootstrap Spot Rates

Price as a Percentage of Par	*Coupon*	*Semiannual Period*	*Maturity (Years)*
102.2969	6.125	1	0.5
104.0469	6.250	2	1.0
104.0000	5.250	3	1.5
103.5469	4.750	4	2.0

Answer:

The spot rates derived by bootstrapping are shown in Figure 4.

Figure 4: Bootstrapped Spot Rate Curve

Price as a Percentage of Par	*Coupon*	*Semiannual Period*	*Maturity (Years)*	*Spot Rates*
102.2969	6.125	1	0.5	1.491%
104.0469	6.250	2	1.0	2.136%
104.0000	5.250	3	1.5	2.515%
103.5469	4.750	4	2.0	2.915%

An alternative verification is to use the spot rates to check if they result in the same prices using the bond valuation equation. For example, using the spot rates will ensure computation of the same price for the 2-year bond:

$$B = \left(\frac{\$4.75}{2} \times e^{-\frac{0.01491}{2}\times 1}\right) + \left(\frac{\$4.75}{2} \times e^{-\frac{0.02136}{2}\times 2}\right) + \left(\frac{\$4.75}{2} \times e^{-\frac{0.02515}{2}\times 3}\right) +$$

$$\left(\$100 + \frac{\$4.75}{2}\right) \times e^{-\frac{0.02915}{2}\times 4} = \$103.5469$$

This results in a bond price of $103.5469. Notice that this is exactly the price of the 2-year bond.

Forward Rates

LO 36.5: Calculate forward interest rates from a set of spot rates.

Forward rates are interest rates implied by the spot curve for a specified future period. The spot rates in Figure 4 are the appropriate rates that an investor should expect to realize for various maturities. Suppose an investor is faced with the following two investments, which are based on the spot curve in Figure 4.

1. Invest for two years at 2.915%.
2. Invest for a year at 2.136% and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$e^{\frac{0.02915}{2}\times 4} = e^{\frac{0.02136}{2}\times 2} \times e^{\frac{R_{Forward}}{2}\times 2}$$

where:
$R_{Forward}$ = the 1-year forward rate one year from now

As we will show, for the two strategies to be equal, $R_{Forward}$ must be 3.693%.

We can simplify this calculation by using the following equation:

$$R_{Forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1)\times\left(\frac{T_1}{T_2 - T_1}\right)$$

where:
R_i = the spot rate corresponding with T_i periods
$R_{Forward}$ = the forward rate between T_1 and T_2

For example, if the 1-year rate is 2.136% and the 2-year rate is 2.915%, the 1-year forward rate one year from now is:

$$R_{Forward} = 0.02915 + (0.02915 - 0.02136)\times\left(\frac{1}{2-1}\right) = 0.03694 = 3.694\%$$

This is the same forward rate (with slight rounding error) that was calculated before.

As a further example, consider the problem of finding the 1-year forward rate three years from now, given a 3-year spot rate of 7.424% and a 4-year spot rate of 8.216% (both continuously compounded annual rates). Based on the previous formula, the continuously compounded 1-year rate three years from now is:

$$0.08216 + (0.08216 - 0.07424)\times\frac{3}{4-3} = 0.10592$$

With this equation, generalizations can be made between the shape of the spot curve and the forward curve. The second term is always positive for an upward-sloping spot curve. Therefore, when there is an upward-sloping spot curve, the corresponding forward rate curve is upward-sloping and above the spot curve. Similarly, when there is a downward-sloping spot curve, the corresponding forward-rate curve is downward-sloping and below the spot curve.

Forward Rate Agreements

LO 36.6: Calculate the value of the cash flows from a forward rate agreement (FRA).

A **forward rate agreement** (FRA) is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$
$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:
L = principal
R_K = annualized rate on L, expressed with compounding period $T_1 - T_2$
R = annualized actual rate, expressed with compounding period $T_1 - T_2$
T_i = time i, expressed in years

The value of an FRA if we're receiving or paying is:

$$\text{value (if receiving } R_K) = L \times (R_K - R_{Forward}) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$
$$\text{value (if paying } R_K) = L \times (R_{Forward} - R_K) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

where:
$R_{Forward}$ = forward rate between T_1 and T_2

Note that R_2 is expressed as a continuously compounded rate.

Example: Computing the payoff from an FRA

Suppose an investor has entered into an FRA where he has contracted to pay a fixed rate of 3% on $1 million based on the quarterly rate in three months. Assume that rates are compounded quarterly. **Compute** the payoff of the FRA if the quarterly rate is 1% in three months.

Answer:

For this FRA, the payoff will take place in six months. The net payoff will be the difference between the fixed-rate payment and the floating rate receipt. If the floating rate is 1% in three months, the payoff at the end of the sixth month will be:

$$\$1{,}000{,}000\ (0.01 - 0.03)(0.25) = -\$5{,}000$$

Example: Computing the value of an FRA

Suppose the 3-month and 6-month LIBOR spot rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of \$5,000,000 between months 3 and 6. **Calculate** the value of the FRA.

Answer:

$$R_{\text{Forward}} = 0.05 + (0.05 - 0.04) \times \left(\frac{1}{2-1}\right) = 0.06 = 6\%$$

$$R_{\text{Forward}}\,(\text{with quarterly compounding}) = 4 \times \left[e^{\frac{0.06}{4}} - 1\right] = 0.060452 = 6.05\%$$

$$\text{value} = \$5{,}000{,}000 \times (0.0800 - 0.0605) \times (0.50 - 0.25) \times e^{-(0.05)(0.5)} = \$23{,}773$$

Duration

LO 36.7: Calculate the duration, modified duration and dollar duration of a bond.

The duration of a bond is the average time until the cash flows on the bond are received. For a zero-coupon bond, this is simply the time to maturity. For a coupon bond, its duration will be necessarily shorter than its maturity. The weights on the time in years until each cash flow is to be received are the proportion of the bond's value represented by each of the coupon payments and the maturity payment. The formula for duration using continuously compounded discounting of the cash flows is:

$$\text{duration} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B}\right]$$

where:
t_i = the time (in years) until cash flow c_i is to be received
y = the continuously compounded yield (discount rate) based on a bond price of B

The usefulness of the duration measure lies in the fact that the approximate change in a bond's price, B, for a parallel shift in the yield curve of Δy is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

The change in yield is often expressed as a **basis point** change. One basis point is equivalent to 0.01%. So a 100 basis point change is a change of 1% in the yield.

Modified duration is used when the yield given is something other than a continuously compounded rate. When the yield is expressed as a semiannually compounded rate, for example, modified duration = duration/(1 + y/2). In general we can express this relation as: modified duration $= \frac{\text{duration}}{1+y/m}$, where m is the number of compounding periods per year.

Note that as m goes to infinity (continuous compounding), the two measures are equal and there is no difference between the two.

On the exam, you may also see a reference to **dollar duration**. Dollar duration is simply modified duration multiplied by the price of the bond.

Convexity

LO 36.8: Evaluate the limitations of duration and explain how convexity addresses some of them.

Duration is a good approximation of price changes for an option-free bond, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors.

In fact, the relationship between bond price and yield is not linear (as assumed by duration) but convex. This convexity shows that the difference between actual and estimated prices widens as the yield swings grow. That is, the widening error in the estimated price is due to the curvature of the actual price path. This is known as the **degree of convexity**.

Fortunately, the amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price. It's important to note that all convexity does is account for the amount of error in the estimated price change based on duration. In other words, it picks up where duration leaves off and converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

Using Convexity to Improve Price Change Estimates

In order to obtain an estimate of the percentage change in price due to convexity, or the amount of price change that is not explained by duration, the following calculation will need to be made:

$$\text{convexity effect} = 1/2 \times \text{convexity} \times \Delta y^2$$

The convexity effect is typically quite small. However, remember that convexity is simply correcting for the error embedded in the duration, so you would expect convexity to have a much smaller effect than duration. Also note that for an option-free bond, the convexity effect is always positive, no matter which direction interest rates move. Thus, for option-free bonds, convexity is always added to duration to modify the price volatility errors embedded in duration. This decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields).

LO 36.9: Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.

By combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in the price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by adding the convexity effect to the duration effect.

Example: Estimating price changes with the duration/convexity approach

Estimate the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

Answer:

Using the duration/convexity approach:

percentage bond price change ≈ duration effect + convexity effect

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01^2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + [(1/2) \times 90 \times (-0.01^2)]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

Theories of the Term Structure

LO 36.10: Compare and contrast the major theories of the term structure of interest rates.

The **expectations theory** suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. In reality, the expectations theory fails to explain all future spot rate expectations. The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The **liquidity preference theory** suggests that most depositors prefer short-term liquid deposits. In order to coax them to lend longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

Key Concepts

LO 36.1

Three types of interest rates are particularly relevant in the interest rate derivative markets: Treasury rates, London Interbank Offered Rate (LIBOR), and repo rates. Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates.

LO 36.2

If we have an initial investment of *A* that earns an annual rate *R*, compounded *m* times a year for *n* years, then it has a future value of:

$$FV = A\left(1+\frac{R}{m}\right)^{m \times n}$$

LO 36.3

In most circumstances, rates are discretely compounded so we need to find the continuously compounded rate that gives the same future value. The continuous rate can be solved as follows:

$$R_c = m \times \ln\left(1+\frac{R}{m}\right)$$

LO 36.4

Zero (spot) rates correspond to the interest earned on a single cash flow at a single point in time. Bond prices are computed using the spot curve by discounting each cash flow at the appropriate spot rate.

The yield of a bond is the single discount rate that equates the present value of a bond to its market price.

Zero rates are computed using the bootstrapping methodology.

LO 36.5

Forward rates are computed from spot rates. When the spot curve is upward-sloping, the corresponding forward rate curve is upward-sloping and above the spot curve. When the spot curve is downward-sloping, the corresponding forward rate curve is downward-sloping and below the spot curve.

LO 36.6

A forward-rate agreement is a contract between two parties that an interest rate will apply to a specific principal during some future time period.

LO 36.7

Duration and modified duration are the same when continuously compounded yields are used, and they both estimate the percentage price change of a bond from an absolute change in yield. Dollar duration is modified duration multiplied by the price of the bond.

LO 36.8

Duration is only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors. The amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price.

LO 36.9

The approximate change in a bond's price, B, for a parallel shift in the yield curve of Δy is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

In order to obtain an estimate of the percentage change in price due to convexity, the following calculation will need to be made:

$$\text{convexity effect} = \frac{1}{2} \times \text{convexity} \times \Delta y^2$$

Combining duration and convexity creates a more accurate estimate of the percentage change in the price of a bond:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

LO 36.10

The expectations theory suggests that forward rates correspond to expected future spot rates. The market segmentation theory states that bonds are segmented into different maturity sectors and that supply and demand dictate rates in the segmented maturity sectors. The liquidity preference theory suggests that longer-term rates incorporate a liquidity premium.

Concept Checkers

1. What is the continuously compounded rate of return for an investment that has a value today of $86.50 and will have a future value of $100 in one year?
   A. 13.62%.
   B. 14.50%.
   C. 15.61%.
   D. 16.38%.

2. Assume that the continuously compounded 10-year spot rate is 5% and the 9-year spot rate is 4.9%. Which of the following is closest to the 1-year forward rate nine years from now?
   A. 4.1%.
   B. 5.1%.
   C. 5.9%.
   D. 6.0%.

3. An investor enters into a 1-year forward rate agreement (FRA) where she will receive the contracted rate on a principal of $1 million. The contracted rate is a 1-year rate at 5%. Which of the following is closest to the cash flow if the actual rate is 6% at maturity of the underlying asset (loan)?
   A. –$10,000.
   B. –$1,000.
   C. +$1,000.
   D. +$10,000.

4. What is the bond price of a $100 face value, 2.5-year, 3% semiannual coupon bond using the following annual continuously compounded spot rates: z_1 = 3%, z_2 = 3.1%, z_3 = 3.2%, z_4 = 3.3%, and z_5 = 3.4%?
   A. $97.27.
   B. $97.83.
   C. $98.15.
   D. $98.99.

5. A $100 face value, 1-year, 4% semiannual bond is priced at 99.806128. If the annualized 6-month spot rate (z_1) is 4.1%, what is the 1-year spot rate (z_2)? (Both spots are continuously compounded rates.)
   A. 4.07%.
   B. 4.16%.
   C. 4.20%.
   D. 4.26%.

Concept Checker Answers

1. B The formula to solve this problem is:

$$R_c = m \times \ln\left(1+\frac{R}{m}\right)$$

First, we need to compute *R* as the rate earned on the \$86.50 investment:

$$R = \frac{\$100 - \$86.50}{\$86.50} = 0.15607$$

This is essentially the effective rate earned over one year with annual compounding. So, m = 1, and R_c = 1 × ln(1.15607) = 0.1450. Alternatively, since m = 1,

$$\ln\left(\frac{100}{86.50}\right) = 0.1450 = 14.50\%$$

2. C $R_{Forward} = R_2 + (R_2 - R_1) \times [T_1 / (T_2 - T_1)] = 0.05 + (0.05 - 0.049) \times [9 / (10 - 9)] = 5.9\%$

3. A \$1,000,000 (0.05 – 0.06)(1) = –\$10,000

4. D $B = 1.5 \times e^{[(-0.03/2)\times 1]} + 1.5 \times e^{[(-0.031/2)\times 2]} + 1.5 \times e^{[(-0.032/2)\times 3]} + 1.5 \times e^{[(-0.033/2)\times 4]} + 101.5 \times e^{[(-0.034/2)\times 5]} = 1.48 + 1.45 + 1.43 + 1.40 + 93.23 = \$98.99$

5. B $B = 2 \times e^{[(-z_1/2)\times 1]} + 102 \times e^{[(-z_2/2)\times 2]}$; $\$99.806128 = 2 \times e^{[(-0.041/2)\times 1]} + 102 \times e^{[(-z_2/2)\times 2]}$; $\$97.846711 = 102 \times e^{[(-z_2/2)\times 2]}$; $z_2 = 0.0415707 = 4.16\%$

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Determination of Forward and Futures Prices

Topic 37

Exam Focus

Both forward and futures contracts are obligations regarding a future transaction. Because the difference in pricing between these contract types is small, forward contract pricing and futures contract pricing are often presented interchangeably. The basic model for forward prices is the cost-of-carry model, which essentially connects the forward price to the cost incurred from purchasing and storing the underlying asset until the contract maturity date. Cash flows over the life of the contract are easily incorporated into the pricing model. Futures contracts contain delivery options that benefit the short seller of the contract. These delivery options must be incorporated into the futures pricing model.

Investment and Consumption Assets

LO 37.1: Differentiate between investment and consumption assets.

An **investment asset** is an asset that is held for the purpose of investing. This type of asset is held by many different investors for the sake of investment. Examples of investment assets include stocks and bonds. A **consumption asset** is an asset that is held for the purpose of consumption. Examples of consumption assets include commodities such as oil and natural gas.

Short-Selling and Short Squeeze

LO 37.2: Define short-selling and calculate the net profit of a short sale of a dividend-paying stock.

Short sales are orders to sell securities that the seller does not own. Short selling is also known as "shorting" and is possible with some investment assets. For a short sale, the short seller (1) simultaneously borrows and sells securities through a broker, (2) must return the securities at the request of the lender or when the short sale is closed out, and (3) must keep a portion of the proceeds of the short sale on deposit with the broker.

The short seller may be forced to close his position if the broker runs out of securities to borrow. This is known as a **short squeeze**, and the seller will need to close his short position immediately.

Why would anyone ever want to sell securities short? The seller thinks the current price is too high and that it will fall in the future, so the short seller hopes to sell high and then buy low. If a short sale is made at $30 per share and the price falls to $20 per share, the short seller can buy shares at $20 to replace the shares borrowed and keep $10 per share as profit.

Two rules currently apply to short selling:

1. The short seller must pay all dividends due to the lender of the security.
2. The short seller must deposit collateral to guarantee the eventual repurchase of the security.

Example: Net profit of a short sale of a dividend-paying stock

Assume that trader Alex Rodgers sold short XYZ stock in March by borrowing 200 shares and selling them for $50/share. In April, XYZ stock paid a dividend of $2/share. **Calculate** the net profit from the short sale assuming Rodgers bought back the shares in June for $40/share in order to replace the borrowed shares and close out his short position.

Answer:

The cash flows from the short sale on XYZ stock are as follows:

March: borrow 200 shares and sell them for $50/share	+$10,000
April: short seller dividend payment to lender of $2/share	–$400
June: buyback shares for $40/share to close short position	–$8,000
Total net profit =	+$1,600

Forward and Futures Contracts

LO 37.3: Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.

LO 37.4: Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices.

LO 37.9: Define and calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows.

Futures contracts and **forward contracts** are *similar* in that both:

- Can be either deliverable or cash settlement contracts.
- Are priced to have zero value at the time an investor enters into the contract.

Futures contracts *differ* from forward contracts in the following ways:

- Futures contracts trade on organized exchanges. Forwards are private contracts and do not trade on an exchange.
- Futures contracts are highly standardized. Forwards are customized contracts satisfying the needs of the parties involved.

- A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
- The government regulates futures markets. Forward contracts are usually not regulated.

Forward Prices

The pricing model used to compute forward prices makes the following assumptions:

- No transaction costs or short-sale restrictions.
- Same tax rates on all net profits.
- Borrowing and lending at the risk-free rate.
- Arbitrage opportunities are exploited as they arise.

For the development of a forward pricing model, we will use the following notation:

- T = time to maturity (in years) of the forward contract.
- S_0 = underlying asset price today ($t = 0$).
- F_0 = forward price today.
- r = continuously compounded risk-free annual rate.

The forward price may be written as:

Equation 1

$$F_0 = S_0e^{rT}$$

The right-hand side of Equation 1 is the cost of borrowing funds to buy the underlying asset and carrying it forward to time *T*. Equation 1 states that this cost must equal the forward price. If $F_0 > S_0e^{rT}$, then arbitrageurs will profit by selling the forward and buying the asset with borrowed funds. If $F_0 < S_0e^{rT}$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in Equation 1 must hold. Note that this model assumes perfect markets.

As it turns out, actual short sales are not necessary for Equation 1 to hold. All that is necessary is a sufficient number of investors who are not only holding the investment asset but also are willing to sell the asset if the forward price becomes too low. In the event that the forward price is too low, the investor will sell the asset and take a long position in the forward contract. This is important since the arbitrage relationship in Equation 1 must hold for all investment assets even though short selling is not available for every asset.

Example: Computing a forward price with no interim cash flows

Suppose we have an asset currently worth $1,000. The current continuously compounded rate is 4% for all maturities. **Compute** the price of a 6-month forward contract on this asset.

Answer:

$$F_0 = \$1{,}000e^{0.04(0.5)} = \$1{,}020.20$$

Forward Price With Carrying Costs

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to Equation 1. Since the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of these cash flows must be deducted from the spot price when calculating the forward price. This is most easily seen when the underlying asset makes a periodic payment. With this in mind, we let I represent the *present value* of the cash flows over T years. Equation 1 then becomes:

Equation 2

$$F_0 = (S_0 - I)\, e^{rT}$$

The same arbitrage arguments used for Equation 1 are used here. The only modification is that the arbitrageur must account for the known cash flows.

Example: Forward price when underlying asset has a cash flow

Compute the price of a 6-month forward on a coupon bond worth \$1,000 that pays a 5% coupon semiannually. A coupon is to be paid in three months. Assume the risk-free rate is 4%.

Answer:

The cost of carry (income) in this case is computed as:

$$I = 25e^{-0.04(0.25)} = \$24.75125$$

Using Equation 2:

$$F_0 = (\$1{,}000 - \$24.75125)e^{0.04(0.5)} = \$994.95$$

The Effect of a Known Dividend

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid continuously. Letting q represent the continuously compounded dividend yield paid by the underlying asset expressed on a per annum basis, Equation 1 becomes:

Equation 3

$$F_0 = S_0 e^{(r-q)T}$$

Once again, the same arbitrage arguments are used to prove that Equation 3 must be true.

Example: Forward price when the underlying asset pays a dividend

Compute the price of a 6-month forward contract for which the underlying asset is a stock index with a value of 1,000 and a continuous dividend yield of 1%. Assume the risk-free rate is 4%.

Answer:

Using Equation 3:

$$F_0 = 1{,}000e^{(0.04-0.01)0.5} = 1{,}015.11$$

Value of a Forward Contract

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other). Since the forward price at every moment in time is computed to prevent arbitrage, the value at inception of the contract must be zero. The forward contract can take on a non-zero value only after the contract is entered into and the obligation to buy or sell has been made. If we denote the obligated delivery price after inception as *K*, then the value of the long contract on an asset with no cash flows is computed as $S_0 - Ke^{-rT}$; with cash flows (with present value *I*) it is $S_0 - I - Ke^{-rT}$; and with a continuous dividend yield of *q*, it is $S_0e^{-qT} - Ke^{-rT}$.

Example: Value of a stock index forward contract

Using the stock index forward in the previous example, **compute** the value of a long position if the index increases to 1,050 immediately after the contract is purchased.

Answer:

In this case, K = 1,015.11 and S_0 = 1,050, so the value is:

$$1{,}050e^{-0.01(0.5)} - 1{,}015.11e^{-0.04(0.5)} = 49.75$$

Currency Futures

LO 37.6: Calculate a forward foreign exchange rate using the interest rate parity relationship.

Interest rate parity (IRP) states that the forward exchange rate, *F* (measured in domestic per unit of foreign currency), must be related to the spot exchange rate, *S*, and to the interest rate differential between the domestic and the foreign country, $r - r_f$.

The general form of the interest rate parity condition is expressed as:

$$F = Se^{(r-r_f)T}$$

This equation is a no-arbitrage relationship. Using our notation from earlier, we can state the interest rate parity relationship as:

Equation 4

$$F_0 = S_0e^{(r-r_f)T}$$

Note that this is equivalent to Equation 3 with r_f replacing q. Just as the continuous dividend yield q was used to adjust the cost of carry, we use the continuous yield on a foreign currency deposit here.

Example: Currency futures pricing

Suppose we wish to **compute** the futures price of a 10-month futures contract on the Mexican peso. Each contract controls 500,000 pesos and is quoted in terms of dollar/peso. Assume that the continuously compounded risk-free rate in Mexico (r_f) is 14%, the continuously compounded risk-free rate in the United States is 2%, and the current exchange rate is 0.12.

Answer:

Applying Equation 4:

$$F_0 = \$0.12e^{(0.02-0.14)\frac{10}{12}} = \$0.10858\,/\,\text{peso}$$

Professor's Note: The concept of interest rate parity will show up again in the foreign exchange risk topic (Topic 45).

Forward Prices vs. Futures Prices

LO 37.5: Explain the relationship between forward and futures prices.

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known over the life of a contract, T, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates. In general, when T is small, the price differences are usually very small and can be ignored. Empirical research comparisons of forwards and futures prices are mixed. Some studies conclude a significant difference and others do not. The important concept to understand here is that assuming the two are the same is an approximation, and under certain circumstances the approximation can be inaccurate.

Commodity Futures

LO 37.7: Define income, storage costs, and convenience yield.

LO 37.8: Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields.

Professor's Note: Topic 44 later in this book is devoted to commodity forwards and futures. In that topic, you will learn more about storage costs and convenience yield as well as the arbitrage relationships that must hold with commodity futures.

Income and Storage Costs

When the underlying is considered a consumption asset, the pricing relationships developed above do not adequately capture all the necessary characteristics of the asset. *Consumption assets have actual storage costs associated with them.* These costs increase the carrying costs. The costs can be expressed either as a known cash flow or as a yield. Let U denote the present value of known storage cost over the life of the forward contract. Equation 1 then becomes:

Equation 5

$$F_0 = (S_0 + U)e^{rT}$$

If we express the storage costs in terms of a continuous yield, u:

Equation 6

$$F_0 = S_0 e^{(r+u)T}$$

The arbitrage relationships are the same except we need to account for the additional carrying costs over T years. However, when the owner of these assets is reluctant to sell the asset, Equations 5 and 6 are replaced by:

Equation 7

$$F_0 \leq (S_0 + U)e^{rT}$$

And:

Equation 8

$$F_0 \leq S_0 e^{(r+u)T}$$

Convenience Yield

Equations 7 and 8 suggest there is a *benefit to owning the underlying consumable asset compared to owning the futures contract.* If we introduce a **convenience yield,** y, to balance Equations 7 and 8, we have:

$$F_0e^{yT} = (S_0 + U)e^{rT} = S_0e^{(r+u)T}$$

This formula can be reduced to:

Equation 9

$$F_0 = S_0e^{(r+u-y)T}$$

In other words, the convenience yield is simply the yield required to produce an equality and is thus a measure of the benefit of owning spot, or physical, consumption commodities.

Delivery Options in the Futures Market

LO 37.10: Describe the various delivery options available in the futures markets and how they can influence futures prices.

Some futures contracts grant **delivery options** to the short—options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

As shown in the previous discussion on commodity futures, if the cost of carrying the asset is greater than the convenience yield (benefit from holding the physical asset), it is ideal for the short position to deliver the contract early. This scenario suggests that the futures price will increase over time; hence, the short has an incentive to deliver early. The opposite relationship holds true when the cost of carry is less than the convenience yield. In this case, the short position will delay delivery since the futures price is expected to fall over time.

Futures and Expected Future Spot Prices

LO 37.11: Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.

The cost of carry model is a widely used method for estimating the appropriate price of a futures contract, but other theories exist for explaining the futures price. One intuitively appealing model expresses the futures price as a function of the expected spot price (S_T).

$$F_0 = E(S_T)$$

For obvious reasons, this is called the **expectations model** and states that the current futures price for delivery at time *T* is equal to the expected spot price at time *T*. Similar to the no-arbitrage rule, this model acts to keep the current futures price in line with the expected spot rate at that time. If the futures price is less than the expected price, aggressive buying of the futures would push up the futures price. If the futures price is greater than the expected spot rate, aggressive selling of the futures would lead to lower the futures price. Although intuitively appealing, other factors probably play a role in the pricing mechanism. Indeed, if the expectations model limited traders to a risk-free rate of return, there would be no incentive to buy or sell contracts.

Cost of Carry vs. Expectations

Economist John Maynard Keynes found the expectations model to be flawed precisely because it provided no justification for speculators to enter the market. Futures contracts provide a mechanism to transfer risk from those who need to hedge their positions (e.g., farmers who are long the commodities) to speculators. In order to entice speculators to bear the risk of these contracts, there has to exist an expectation of profit greater than the risk-free rate. For this to occur, the futures contract price must be less than the expected spot rate at maturity [$F_0 < E(S_T)$] and must continually increase during the term of the contract. Keynes referred to this as **normal backwardation.** This relationship suggests that the asset underlying the futures contract exhibits positive systematic risk, since this is the risk that remains after diversifying away all nonsystematic risk.

On the other side of the contracts are those who are users of the commodity who want to shift some of the risk of rising market prices to speculators. They wish to purchase futures contracts from speculators. The speculators have to be enticed into assuming this risk by the expectation of profits that would exceed the risk-free rate. From this perspective, the futures price must be higher than the expected spot price at maturity [$F_0 > E(S_T)$] and must continually decrease during the term of the contract. Keynes referred to this expectation as **contango** (a.k.a. normal contango). This relationship suggests that the asset underlying the futures contract exhibits negative systematic risk.

Contango and Backwardation

LO 37.12: Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model.

Backwardation refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset. Backwardation might occur if there are benefits to holding the asset that offset the opportunity cost of holding the asset (the risk-free rate) and additional net holding costs.

Contango refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.

Professor's Note: In this case, the reference to backwardation and contango refers to the relationship between the futures price and the current spot price, not the expected spot price.

Key Concepts

LO 37.1

An investment asset is an asset that is held for the purpose of investing. A consumption asset is an asset that is held for the purpose of consumption.

LO 37.2

Short sales are orders to sell securities that the seller does not own. A short squeeze results if the broker runs out of securities to borrow.

LO 37.3

Forward and futures contracts are similar because they are both future obligations to transact an asset on some future date. Forward contracts do not trade on an exchange, are not standardized, and do not normally close out prior to expiration.

The relationship between forward and spot prices is as follows:

$$F = S_0 e^{rT}$$

LO 37.4

The cost-of-carry model is used to price forward and futures contracts. It states that the total cost of carrying the underlying asset to expiration must be the futures price. Any other price results in arbitrage.

LO 37.5

When interest rates are known over the life of a contract, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates.

LO 37.6

Interest rate parity states that the forward exchange rate, F (measured in domestic per unit of foreign currency), must be related to the spot exchange rate, S, and to the interest rate differential between the domestic and the foreign country:

$$F = S_0 e^{(r_{DC} - r_{FC})T}$$

LO 37.7

Consumption assets have actual storage costs (known as carrying costs) associated with them.

If there is a benefit to owning the underlying consumable asset compared to owning the futures, the futures price will incorporate a convenience yield.

LO 37.8

Futures price with storage costs, *u*: $F = S_0e^{(r+u)T}$

Futures price with convenience yield, *y*: $F = S_0e^{(r+u-y)T}$

LO 37.9

The futures price or cost-of-carry model is easily accommodated for interim cash flows from the underlying asset. If the underlying asset pays a known amount of cash, *I*, over the life of the forward contract, a simple adjustment is made to the cost-of-carry model:

$$F = (S_0 - I)e^{rT}$$

When the underlying asset pays a dividend, *q*, we assume that the dividend is paid continuously:

$$F = S_0e^{(r-q)T}$$

LO 37.10

Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract. Futures contracts are typically "offset" by buying or selling a contract before the delivery date. Only a small percentage of contracts result in physical delivery.

LO 37.11

The expectations model states that the current futures price for delivery at time *T* is equal to the expected spot price at time *T*. This model acts to keep the current futures price in line with the expected spot rate at that time.

LO 37.12

Contango is the situation in which the futures price is above the current spot price. Backwardation is the opposite relationship.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An investor has an asset that is currently worth \$500, and the continuously compounded risk-free rate at all maturities is 3%.

1. Which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
   A. \$496.26.
   B. \$500.00.
   C. \$502.00.
   D. \$503.76.

2. If the asset pays a continuous dividend of 2%, which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
   A. \$494.24.
   B. \$498.75.
   C. \$501.25.
   D. \$506.29.

3. A bond pays a semiannual coupon of \$40 and has a current value of \$1,109. The next payment on the bond is in four months and the interest rate is 6.50%. Using the continuous time model, the price of a 6-month forward contract on this bond is closest to:
   A. \$995.62.
   B. \$1,011.14.
   C. \$1,035.65.
   D. \$1,105.20.

4. The owner of 300,000 bushels of corn wishes to hedge his position for a sale in 150 days. The current price of corn is \$1.50/bushel and the contract size is 5,000 bushels. The interest rate is 7%, compounded daily. The storage cost for the corn is \$18/day. Assume the cost of storage as a percentage of the contract per year is 1.46%. The price for the appropriate futures contract used to hedge the position is closest to:
   A. \$6,635.
   B. \$7,248.
   C. \$7,656.
   D. \$7,765.

5. Backwardation refers to a situation where:
   A. spot prices are above futures prices.
   B. spot prices are below futures prices.
   C. expected future spot prices are above futures prices.
   D. expected future spot prices are below futures prices.

Concept Checker Answers

1. D Using Equation 1:

$$500e^{(0.03)(0.25)} = \$503.76$$

where S = 500, T = 0.25, and r = 0.03

2. C Using Equation 3:

$$500e^{(0.03-0.02)0.25} = \$501.25$$

3. D Use the formula $F_0 = (S_0 - I)e^{rT}$, where *I* is the present value of $40 to be received in 4 months, or 0.333 years. At a discount rate of 6.50%:

$$I = \$40 \times e^{-0.065 \times 0.333} = \$39.14$$

$$F_0 = (\$1{,}109 - 39.14) \times e^{(0.065 \times 0.5)} = \$1{,}105.20$$

4. D Since both the interest and the storage costs compound on a daily basis, a continuous time model is appropriate to approximate the price of the contract.

The cost of storage as a percentage of the contract per year is:

$$u = 365 \times \frac{18}{1.50 \times 300{,}000} = 0.0146$$

Using Equation 6, the futures price per bushel is:

$$F = \$1.50 \times e^{(0.07 + 0.0146)(150/365)} = \$1.553 \times 5{,}000 \text{ bushels per contract} = \$7{,}765.34$$

5. A Backwardation refers to a situation where spot prices are higher than futures prices. Significant monetary benefits of the asset or a relatively high convenience yield can lead to this result.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Interest Rate Futures

Topic 38

Exam Focus

In this topic, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. Be able to define the cheapest-to-deliver bond for T-bonds and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. Be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

Day Count Conventions

LO 38.1: Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual.**
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

The following examples demonstrate the use of day count conventions when computing accrued interest.

Example: Day count conventions

Suppose there is a semiannual-pay bond with a $100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. **Compute** the accrued interest of this bond as a T-bond and a U.S. corporate bond.

Answer:

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 (= 30 × 4 + 12) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

Quotations for T-Bonds

LO 38.3: Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.

T-bond prices are quoted relative to a $100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

cash price = quoted price + accrued interest

Clean and Dirty Prices

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. This relationship is shown in the equation above. Conversely, the clean price is the cash price less accrued interest:

quoted price = cash price – accrued interest

This relationship can also be expressed as:

clean price = dirty price – accrued interest

Example: Calculate the cash price of a bond

Assume the bond in the previous example is a T-bond currently quoted at 102–11. **Compute** the cash price.

Answer:

cash price = \$102.34375 + \$2.1848 = \$104.52855

For a \$100,000 par amount, this is \$104,528.55.

Quotations for T-Bills

LO 38.2: Calculate the conversion of a discount rate to a price for a US Treasury bill.

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a \$100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

Example: Calculating the cash price on a T-bill

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is \$100, what is the true rate of interest and the cash price?

Answer:

Interest is equal to \$2.5 (= \$100 × 0.05 × 180 / 360) for a 180-day period. The true rate of interest for the period is therefore 2.564% [= 2.5 / (100 – 2.5)].

Cash price: 5 = (360 / 180) × (100 – Y); Y = \$97.5.

Treasury Bond Futures

LO 38.4: Explain and calculate a US Treasury bond futures contract conversion factor.

LO 38.5: Calculate the cost of delivering a bond into a Treasury bond futures contract.

LO 38.6: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.

In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. This produces a large supply of potential bonds that are deliverable on the contract and reduces the likelihood of market manipulation. Since the deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

cash received = (QFP × CF) + AI

where:
QFP = quoted futures price (most recent settlement price)
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as: (discounted price of a bond – accrued interest) / face value. For example, if the present value of a bond is $142, accrued interest is $2, and face value is $100, the conversation factor would be: (142 – 2) / 100 = 1.4.

Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

cash received by the short = (QFP × CF) + AI

cost to purchase bond = (quoted bond price + AI)

The CTD bond minimizes the following: quoted bond price – (QFP × CF). This expression calculates the cost of delivering the bond.

Example: The cheapest-to-deliver bond

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is $95.75 (this is the quoted futures price). **Determine** which bond is the cheapest-to-deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

Answer:

Cost of delivery:

Bond 1: 99 – (95.75 × 1.01) = $2.29

Bond 2: 125 – (95.75 × 1.24) = $6.27

Bond 3: 103 – (95.75 × 1.06) = $1.51

Bond 4: 115 – (95.75 × 1.14) = $5.85

Bond 3 is the cheapest-to-deliver with a cost of delivery of $1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest-to-deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

Treasury Bond Futures Price

LO 38.7: Calculate the theoretical futures price for a Treasury bond futures contract.

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous topic. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I)e^{rT}$$

where:
I = present value of cash flow

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

Example: Theoretical futures price

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. **Calculate** the theoretical price for this T-bond futures contract.

Answer:

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$AI = \text{coupon} \times \left(\frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}} \right)$$

$$AI = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Since the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation which discounts the cash flow using the risk-free rate:

$$5e^{-0.03 \times (90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, since the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price – AI).

$$98.99 - \left(5 \times \frac{90}{180}\right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$QFP = \frac{96.49}{1.1} = \$87.72$$

Eurodollar Futures

LO 38.8: Calculate the final contract price on a Eurodollar futures contract.

LO 38.9: Describe and compute the Eurodollar futures contract convexity adjustment.

The 3-month **eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one "tick," which is a price change of one basis point, or $25 per $1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of $1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR. If Z is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10{,}000[100 - (0.25)(100 - Z)]$$

For example, if the quoted price, Z, is 97.8:

$$\text{contract price} = \$10{,}000[100 - (0.25)(100.0 - 97.8)] = \$994{,}500$$

Convexity Adjustment

The corresponding 90-day forward LIBOR (on an annual basis) for each contract is 100 – Z. For example, assume that the previous eurodollar contract was for a futures contract that matured in six months. Then the 90-day forward LIBOR six months from now is approximately 2.2% (100 – 97.8). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

$$\text{actual forward rate} = \text{forward rate implied by futures} - (\tfrac{1}{2} \times \sigma^2 \times T_1 \times T_2)$$

where:
T_1 = the maturity on the futures contract
T_2 = the time to the maturity of the rate underlying the contract (90 days)
σ = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as T_1 increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the σ and the T_2 are largely dictated by the specifications of the futures contract.

LO 38.10: Explain how Eurodollar futures can be used to extend the LIBOR zero curve.

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve (also called a LIBOR zero curve since spot rates are sometimes referred to as zero rates). Recall the equation presented previously in Topic 36, which was used to generate the shape of the *futures* rate curve:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i periods
R_{Forward} = the forward rate between T_1 and T_2

This forward rate equation can be rearranged to solve for the *spot* rate for the next time period (T_2):

$$R_2 = \frac{R_{\text{Forward}}(T_2 - T_1) + R_1 T_1}{T_2}$$

Given the first LIBOR spot rate (R_1) and the length of each forward contract period, we can calculate the next spot rate (R_2). The rate at T_2 can then be used to find the rate at T_3 and so on. The end result is a generated LIBOR spot (zero) curve.

Duration-Based Hedging

LO 38.11: Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.

The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of *P* and a futures position with a contract value of *F*. Denote the duration of the portfolio at the hedging horizon as D_P and the corresponding duration of the futures contract as D_F. Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:
N = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short *N* contracts to produce a position with a zero duration.

Example: Duration-based hedge

Assume there is a 6-month hedging horizon and a portfolio value of $100 million. Further assume that the 6-month T-bond contract is quoted at 105–09, with a contract size of $100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. **Outline** the appropriate hedge for small changes in yield.

Answer:

$$N = -\frac{100,000,000 \times 10}{105,281.25 \times 12} = -791.53$$

Rounding up to the nearest whole number means the manager should short 792 contracts.

Limitations of Duration

LO 38.12: Explain the limitations of using a duration-based hedging strategy.

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

Key Concepts

LO 38.1

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date. The most common day count conventions are Actual/Actual, 30/360, and Actual/360.

LO 38.2

T-bills are quoted on a discount rate basis. A T-bill with a $100 face value with *n* days to maturity and a cash price of *Y* is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

LO 38.3

For a U.S. Treasury bond, the dirty price is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond plus the accrued interest. Conversely, the clean price is the dirty price less accrued interest.

LO 38.4

Since deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created conversion factors. Conversion factors are supplied by the CBOT on a daily basis. They are calculated as:

(bond discounted price – accrued interest) / face value

LO 38.5

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The cheapest-to-deliver (CTD) bond is the bond that minimizes the following:

quoted bond price – (quoted futures price × conversion factor)

LO 38.6

When the yield curve is not flat, there is a single bond that is the cheapest-to-deliver (CTD). When the yield curve is upward sloping, CTD bonds tend to have longer maturities. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

LO 38.7

The theoretical price for a T-bond futures contract is calculated as:

(cash futures price – accrued interest) / conversion factor

LO 38.8

Eurodollar contracts are based on LIBOR and are quoted on a discount rate basis. If *Z* is the quoted price for a eurodollar futures contract, the contract price is:

eurodollar futures price = $10,000 × [100 – (0.25) × (100 – Z)]

LO 38.9

Long-dated eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates. As the maturity of the futures contract increases, the necessary convexity adjustment increases.

LO 38.10

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve. The following equation is used to generate the shape of the futures rate curve:

$$R_{\text{Forward}} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i periods
R_{Forward} = the forward rate between T_1 and T_2

LO 38.11

Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = -\frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

LO 38.12

The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

Concept Checkers

1. Assume a 6-month hedging horizon and a portfolio value of $30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100–13, with a contract size of $100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
   A. Long 298 contracts.
   B. Short 298 contracts.
   C. Long 199 contracts.
   D. Short 199 contracts.

2. Which of the following items limits the use of duration as a risk metric?
   I. It assumes the price/yield relationship is linear.
   II. It assumes interest rate volatility is constant.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Consider day count convention and, specifically, the following example: A semiannual bond with $100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is true?
   A. Corporate bonds accrue more interest in July than T-bonds.
   B. Corporate bonds accrue more interest from March 1 to September 1 than September 1 to March 1.
   C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
   D. The T-bond accrued interest is $1.76 for this period (March 1 to August 3).

4. Assume an investor is about to deliver a short bond position and has four options to choose from which are listed in the following table. The settlement price is $92.50 (i.e., the quoted futures price). Determine which bond is the cheapest-to-deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

   A. Bond 1.
   B. Bond 2.
   C. Bond 3.
   D. Bond 4.

5. Assume the cash price on a 90-T-bill is quoted as 98.75. The discount rate is closest to:
   A. 4%.
   B. 7%.
   C. 6%.
   D. 5%.

Concept Checker Answers

1. D $N = -\frac{(\$30{,}000{,}000 \times 8)}{(\$100{,}406.25 \times 12)} = -199$

   The appropriate hedge is to short 199 contracts.

2. A The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes the price/yield relationship is linear, and (3) it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated).

3. C July accrued T-bond interest is 31/184 = 0.1685; July accrued corporate bond interest is 30/180 = 0.1667. T-bonds accrue 155/184 = 0.8424 × \$2 = \$1.6848; C-bonds accrue 152/180 = 0.8444 × \$2 = \$1.6889.

4. A Cost of delivery:

   Bond 1: 98 – (92.50 × 1.02) = \$3.65

   Bond 2: 122 – (92.50 × 1.27) = \$4.53

   Bond 3: 105 – (92.50 × 1.08) = \$5.10

   Bond 4: 112 – (92.50 × 1.15) = \$5.63

   Bond 1 is the cheapest-to-deliver with a cost of delivery of \$3.65.

5. D The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Swaps

Topic 39

Exam Focus

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

Mechanics of Interest Rate Swaps

LO 39.1: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

The most common interest rate swap is the **plain vanilla interest rate swap.** In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 1. The fixed rate of the swap is 3.784%, and the notional principal is $100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 1: 6-Month LIBOR

Beginning of Period	*LIBOR*
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of period one and uses the LIBOR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

$$\text{floating} = \$100 \text{ million} \times 0.03 \times 0.5 = \$1.5 \text{ million}$$

$$\text{fixed} = \$100 \text{ million} \times 0.03784 \times 0.5 = \$1.892 \text{ million}$$

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of $0.392 million. Note that we are ignoring the many day-count and business-day conventions associated with swaps. Figure 2 shows the other cash flows.

Figure 2: Swap Cash Flows

End of Period	*LIBOR at Beginning of Period*	*Floating Cash Flow*	*Fixed Cash Flow*	*Net X Cash Flow*
1	3.00%	$1,500,000	$1,892,000	–$392,000
2	3.50%	$1,750,000	$1,892,000	–$142,000
3	4.00%	$2,000,000	$1,892,000	$108,000
4	4.50%	$2,250,000	$1,892,000	$358,000

LO 39.2: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to LIBOR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

Financial Intermediaries

LO 39.3: Explain the role of financial intermediaries in the swaps market.

LO 39.4: Describe the role of the confirmation in a swap transaction.

In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.

- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swap market participants.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms, act as principals in trades just as they do in forward contracts. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap since both parties will likely only transact with the intermediary. **Financial intermediaries,** such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor and fixed/floating rates) and the steps taken in the event of default.

Comparative Advantage

LO 39.5: Describe the comparative advantage argument for the existence of interest rate swaps and explain some of the criticisms of this argument.

Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 3.

Figure 3: Borrowing Rates for X and Y

Company	*Fixed Borrowing*	*Floating Borrowing*
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at LIBOR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on LIBOR. If we assume the net savings is split evenly, the net borrowing costs for X are then 6.2% and LIBOR – 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

Problems With Comparative Advantage

A problem with the comparative advantage argument is that it assumes X can borrow at LIBOR + 1% over the life of the swap. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

Valuing Interest Rate Swaps

The Discount Rate

LO 39.6: Explain how the discount rates in a plain vanilla interest rate swap are computed.

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce a LIBOR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists when continuous compounding is used:

$$R_{forward} = R_2 + (R_2 - R_1)\,\frac{T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i years
$R_{forward}$ = forward rate between T_1 and T_2

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

Valuing an Interest Rate Swap With Bonds

LO 39.7: Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.

Let's return to our two companies, X and Y, in our 2-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as B_{fix} and the present value of the floating-leg payments as B_{flt}, the value of the swap can be written for both X and Y as:

$$V_{swap}(X) = B_{flt} - B_{fix}$$

$$V_{swap}(Y) = B_{fix} - B_{flt}$$

Note that $V_{swap}(X) + V_{swap}(Y) = 0$. This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that $V_{swap}(X) = V_{swap}(Y) = 0$. As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Since $V_{swap} = \text{Bond}_{fixed} - \text{Bond}_{floating}$, we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

Example: Valuing an interest rate swap

Consider a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the bond methodology.

Answer:

$$B_{fixed} = \left(PMT_{fixed,\,3\text{ months}} \times e^{-(r \times t)}\right) + \left(PMT_{fixed,\,9\text{ months}} \times e^{-(r \times t)}\right) + \left[\left(\text{notional} + PMT_{fixed,\,15\text{ months}}\right) \times e^{-(r \times t)}\right]$$

$$B_{fixed} = \left(\$30{,}000 \times e^{-(0.054 \times 0.25)}\right) + \left(\$30{,}000 \times e^{-(0.056 \times 0.75)}\right) + \left[\left(\$1{,}000{,}000 + \$30{,}000\right) \times e^{-(0.058 \times 1.25)}\right]$$
$$= \$29{,}598 + \$28{,}766 + \$957{,}968 = \$1{,}016{,}332$$

$$B_{floating} = \left[\text{notional} + \left(\text{notional} \times \frac{r_{floating}}{2}\right)\right] \times e^{-(r \times t)}$$
$$= \left[\$1{,}000{,}000 + \left(\$1{,}000{,}000 \times \frac{0.05}{2}\right)\right] \times e^{-(0.054 \times 0.25)} = \$1{,}011{,}255$$

$$V_{swap} = \left(B_{fixed} - B_{floating}\right) = \$1{,}016{,}332 - \$1{,}011{,}255 = \$5{,}077$$

Figure 4 sums up the payments and present value factors.

Figure 4: Valuing an Interest Rate Swap With Two Bond Positions

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

* Note that some rounding has occurred.

Again we see that the value of the swap = 1,016,332 – 1,011,255 = $5,077.

Valuing an Interest Rate Swap With FRAs

LO 39.8: Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).

At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as LIBOR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

Example: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the FRA methodology.

Answer:

To calculate the value of the swap, we'll need to find the floating rate cash flows by calculating the expected forward rates via the LIBOR based spot curve.

The first floating rate cash flow is calculated in a similar fashion to the previous example.

LIBOR rate (last payment date): 5%.

Floating rate cash flow in 3 months: 1,000,000 × 0.05 / 2 = $25,000.

The second floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned forward rate formula:

$$R_{\text{forward}} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

$$R_{\text{forward}} = 0.056 + (0.056 - 0.054)\frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

This rate is a continuously compounded rate, so we need to find the equivalent forward rate with semiannual compounding:

$R_{\text{forward (SC)}} = 2 \times [e^{(0.057/2)} - 1] = 0.05782 = 5.782\%$

Floating rate cash flow in 9 months: 1,000,000 × 0.05782 / 2 = $28,910

The third floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{\text{forward}} = 0.058 + (0.058 - 0.056)\frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

$R_{\text{forward (SC)}} = 2 \times [e^{(0.061/2)} - 1] = 0.06194 = 6.1939\%$

Floating rate cash flow in 15 months: 1,000,000 × 0.061939 / 2 = $30,969

Figure 5: Valuing an Interest Rate Swap Based on a Sequence of FRAs

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	25,000	0.9866*	29,598	24,665
0.75 (9 months)	30,000	28,910	0.9589*	28,766	27,721
1.25 (15 months)	30,000	30,969	0.9301*	27,902	28,803
Total				86,266	81,189

* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs = 86,266 – 81,189= $5,077.

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces the same result as valuing a swap based on two simultaneous bond positions.

Currency Swaps

LO 39.9: Explain the mechanics of a currency swap and compute its cash flows.

LO 39.11: Calculate the value of a currency swap based on two simultaneous bond positions.

A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD175 million, and B pays GBP100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap since both legs of the swap are fixed. Every period (12 months), A will pay GBP6 million to B, and B will pay USD8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (since it's an inflow) and a short position in a GBP-denominated note (since it's an outflow).

If we denote the present value of the GBP-denominated payments as B_{GBP} and the present value of the USD payments as B_{USD}, the value of the swap in USD to Company A is:

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

where:
S_0 = spot rate in USD per GBP

Example: Calculate the value of a currency swap

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. **Value** the currency swap just discussed assuming the swap will last for three more years.

Answer:

$$B_{USD} = 8.75e^{-0.02\times1} + 8.75e^{-0.02\times2} + 183.75e^{-0.02\times3} = \text{USD}190.03 \text{ million}$$

$$B_{GBP} = 6e^{-0.04\times1} + 6e^{-0.04\times2} + 106e^{-0.04\times3} = \text{GBP}105.32 \text{ million}$$

$$V_{swap} \text{ (to A in USD)} = 190.03 - (1.5 \times 105.32) = \text{USD}32.05 \text{ million}$$

LO 39.12: Calculate the value of a currency swap based on a sequence of FRAs.

The value of a currency swap can also be calculated based on a sequence of FRAs.

Example: Value of a currency swap with FRAs

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1.

Compute the value of the currency swap discussed previously using a sequence of FRAs to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

Figure 6: Forward Rates

Year 1	\$1.47/£
Year 2	\$1.44/£
Year 3	\$1.41/£

Professor's Note: The year 1 forward rate is calculated as follows: $F_1 = 1.5e^{(0.02-0.04)\times1} = \$1.47/£$. *Interest rate parity suggests that the dollar will appreciate relative to the pound, so the \$/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP). We will discuss interest rate parity in the foreign exchange risk topic (Topic 45).*

Answer:

Figure 7 denotes the cash flows and forward rates for this currency swap.

Figure 7: Valuing a Currency Swap Based on a Sequence of FRAs

Time	*USD Cash Flow*	*GBP Cash Flow*	*Forward Rate*	*\$ Value of £*	*Net Cash Flows*	*PV of Net CF*
1	8.75	6	1.47	8.82	–0.07	–0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
3	175	100	1.41	141	34	32.02
Total						32.33*

* Note some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is 32 million using both the two simultaneous bond positions and the forward rate agreements.

Using a Currency Swap to Transform Existing Positions

LO 39.10: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate.

Comparative Advantage

Comparative advantage is also used to explain the success of currency swaps. Typically, a domestic borrower will have an easier time borrowing in his own currency. This often results in comparative advantages that can be exploited by using a currency swap. The argument is directly analogous to that used for interest rate swaps. Suppose A and B have the 5-year borrowing rates in the United States and Germany (EUR) shown in Figure 8.

Figure 8: Borrowing Rates

Borrowing Rates for A and B		
Company	*USD Borrowing*	*EUR Borrowing*
A	5.0%	7.0%
B	6.0%	7.5%

Company A needs EUR, and Company B needs USD. Company A has an absolute advantage in both markets but a comparative advantage in the USD market. Notice that the differential between A and B in the USD market is 1%, or 100 basis points (bps), and the corresponding differential in the EUR market is only 50 basis points. When this is the case, A has a comparative advantage in the USD market, and B has a comparative advantage in the EUR market. The net potential borrowing savings by entering into a swap is the difference between the differences, or 50 bps. In other words, by entering into a currency swap, the savings for both A and B totals 50 bps.

Swap Credit Risk

LO 39.13: Describe the credit risk exposure in a swap position.

Because $V_{swap}(A) + V_{swap}(B) = 0$, whenever one side of a swap has a positive value, the other side must be negative. For example, if $V_{swap}(A) > 0$, $V_{swap}(B) < 0$. As $V_{swap}(A)$ increases in value, $V_{swap}(B)$ must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

Other Types of Swaps

LO 39.14: Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of the period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

As you can see, many different types of swaps exist. Some additional examples include: accrual swaps, cancelable swaps, index amortizing rate swaps, and constant maturity swaps. Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

Key Concepts

LO 39.1

A plain vanilla interest rate swap exchanges floating-rate payments (LIBOR) for fixed-rate payments over the life of the swap. The floating rate payments at time t in a plain vanilla interest rate swap are computed using the floating rate at time t – 1.

LO 39.2

Interest rate swaps can be combined with existing asset and liability positions to drastically change the interest rate risk.

LO 39.3

A swap dealer or financial intermediary facilitates the ability to enter into swaps.

LO 39.4

Confirmations outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details and the steps taken in the event of default.

LO 39.5

The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.

LO 39.6

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The cash flows are discounted using the corresponding spot rate from the LIBOR spot curve.

LO 39.7

The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.

LO 39.8

Valuing a swap based on a sequence of forward rate agreements (FRAs) produces the same result as valuing a swap based on two simultaneous bond positions.

LO 39.9

A currency swap exchanges interest rate payments in two different currencies. The exchange rate used in currency swaps is the spot exchange rate.

LO 39.10

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset.

LO 39.11

Since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in an interest rate swap.

LO 39.12

In addition to valuing a currency swap based on two simultaneous bond positions, the value of a currency swap can also be calculated based on a sequence of FRAs.

LO 39.13

Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.

LO 39.14

Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and currency swaps, include: equity swaps, commodity swaps, and volatility swaps.

Concept Checkers

Use the following data to answer Question 1.

Two companies, C and D, have the borrowing rates shown in the following table.

Borrowing Rates for C and D

Company	*Fixed Borrowing*	*Floating Borrowing*
C	10%	LIBOR + 50 bps
D	12%	LIBOR + 100 bps

1. According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?
   A. 0.5%.
   B. 1.0%.
   C. 1.5%.
   D. 2.0%.

2. Which of the following is most accurate regarding the credit risk of a currency swap? As the value of the:
   I. domestic currency leg increases, so does the credit risk of the domestic currency payer.
   II. foreign currency leg increases, so does the credit risk of the foreign currency payer.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following would properly transform a floating-rate liability to a fixed-rate liability? Enter into a pay:
   A. foreign currency swap.
   B. fixed interest rate swap.
   C. domestic currency swap.
   D. floating interest rate swap.

4. Use the following information to determine the value of the swap to the floating rate payer using the bond methodology. Assume we are at the floating rate reset date.
   - $1 million notional value, semiannual, 18-month maturity.
   - Spot LIBOR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
   - The fixed rate is 2.8%, with semiannual payments.

   A. –$66.
   B. $476.
   C. $3,425.
   D. $5,077.

5. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of $150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is $1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for two more years?
   A. $3.53 million.
   B. $52.98 million.
   C. $8.09 million.
   D. $12.74 million.

Concept Checker Answers

1. C The difference of the differences is (12% – 10%) – [LIBOR + 1% – (LIBOR + 0.5%)] = 1.5%.

2. D As one currency (A) appreciates relative to another currency (B), the value of a currency swap increases on behalf of the currency A payer. As a result, the credit risk of the currency B payer increases.

3. B The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability.

4. B $B_{fix} = [\$14{,}000 \times e^{-(0.026\times0.5)}] + [\$14{,}000 \times e^{-(0.0265\times1.0)}] + [(\$1{,}000{,}000 + \$14{,}000) \times e^{-(0.0275\times1.5)}] = \$13{,}819 + \$13{,}634 + \$973{,}023 = \$1{,}000{,}476$

   Note that we are at a (semiannual) reset date, so the floating rate portion has a value equal to the notional amount.

   $V_{swap} = (B_{fix} - B_{floating}) = \$1{,}000{,}476 - \$1{,}000{,}000 = \$476$

5. C $B_{\$} = 6e^{-0.03\times1} + 156e^{-0.03\times2} = \$5.82 + \$146.92 = \$152.74$

   $B_{€} = 5e^{-0.05\times1} + 105e^{-0.05\times2} = €4.76 + €95.00 = €99.76$

   V_{swap} (to X) = 152.74 – (1.45 × 99.76) = \$8.09 million

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Mechanics of Options Markets

Topic 40

Exam Focus

Stock options give the owner the right, but not the obligation, to buy or sell a stock at a specific price on or before a specific date. Call options give the owner the right to buy the stock, and put options give the owner the right to sell the stock. An option is exercised when the owner executes the right to buy or sell the stock. This topic covers the basic mechanics of option trading. You should understand the different kinds of options and the system by which exchange-traded options are bought and sold.

Option Types

LO 40.1: Describe the types, position variations, and typical underlying assets of options.

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. *American options* may be exercised at any time up to and including the contract's expiration date, while *European options* can be exercised only on the contract's expiration date.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

- X = strike price or exercise price specified in the option contract (a fixed value)
- S_t = price of the underlying asset at time t
- C_t = the market value of a call option at time t
- P_t = the market value of a put option at time t
- t = the time subscript, which can take any value between 0 and T, where T is the maturity or expiration date of the option

Call Options

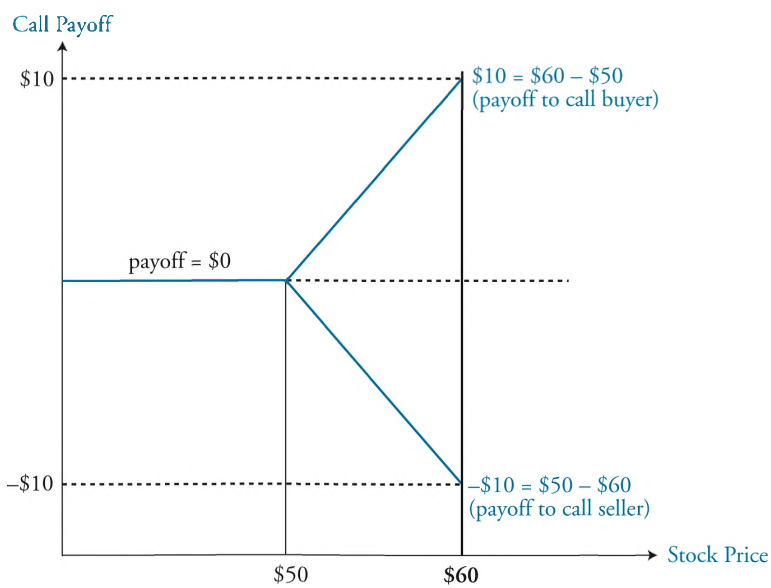
A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

At maturity time T, if the price of the underlying stock is less than or equal to the strike price of a call option (i.e., $S_T \leq X$), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e., $S_T > X$) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ($S_T - X$). The "payoff" (at the option's maturity) to the call option seller, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.

Professor's Note: An option payoff graph ignores the initial cost of the option.

Figure 1: Payoff of Call With Exercise Price Equal to $50



Example: Payoff of a call option

An investor writes an at-the-money call option on a stock with an exercise price of 50 ($X = 50$). If the stock price rises to \$60, what will be the *payoff* to the owner and seller of the call option?

Answer:

The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a \$10 gain. The payoff to the option buyer is \$10, and the payoff to the option writer is *negative* \$10. This is illustrated in Figure 1 and, as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time T (the option's maturity), the payoff to the option owner, represented by C_T is:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$

$$C_T = 0 \quad \text{if} \quad S_T \leq X$$

Another popular way of writing this is with the "max (0, variable)" notation. If the variable in this expression is greater than zero, then max (0, variable) = variable; if the variable's value is less than zero, then max (0, variable) = 0. Thus, letting the variable be the quantity $S_0 - X$, we can write:

$$C_T = \max(0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that max $(0, S_t - X)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the call option early. In this topic, we will only consider time T in our analysis.

Although our focus here is not to calculate C_t, we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is T units of time before T. C_0 is also called the premium. Thus, we can write that the profit to the owner at $t = T$ is:

$$\text{profit} = C_T - C_0$$

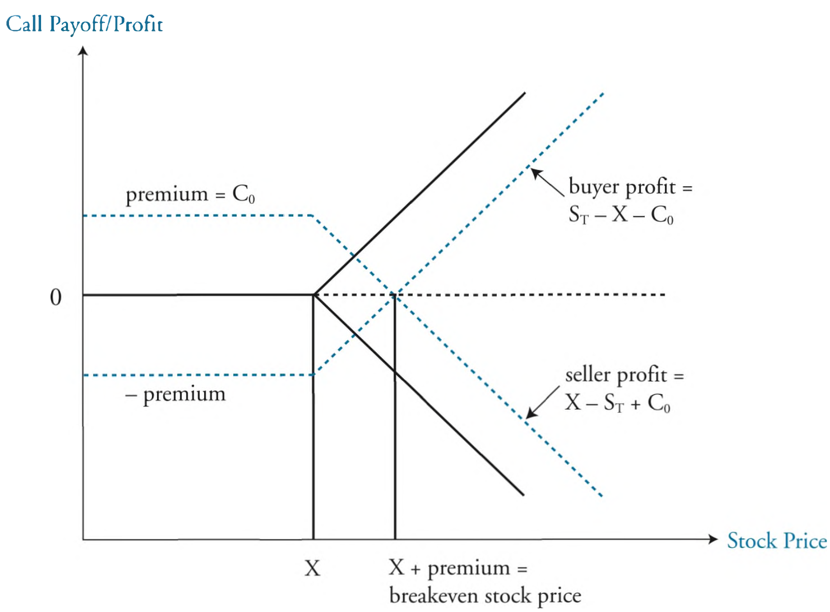
This says that at time T, the owner's profit is the option payoff minus the premium paid at time 0. Incorporating C_0 into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.

Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

if $S_T < X + C_0$ then: call buyer profit < 0 < call seller profit
if $S_T = X + C_0$ then: call buyer profit = 0 = call seller profit
if $S_T > X + C_0$ then: call buyer profit > 0 > call seller profit

The **breakeven price** is a very descriptive term that we use for $X + C_0$, or X + premium.

Figure 2: Profit Diagram for a Call at Expiration



Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price *X*:

$P_T = X - S_T$ if $S_T < X$
$P_T = 0$ if $X \leq S_T$

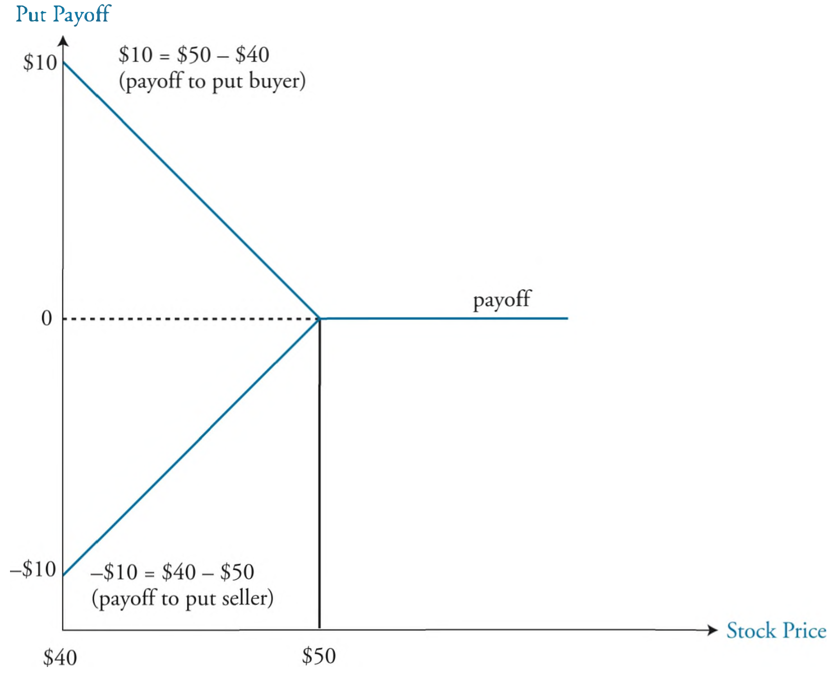
or:

$P_T = \max(0, X - S_T)$

For example, an investor writes a put option on a stock with a strike price of X = 50. If the stock stays at \$50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below \$50, say to \$40, the put option may

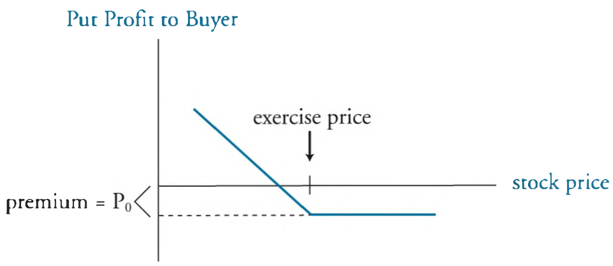
be exercised with the option holder buying the stock from the market at $40 and selling it to the put writer at $50 for a $10 gain. The writer of the put option must pay the put price of $50 when it can be sold in the market at only $40, resulting in a $10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 3: Put Payoff to Buyer and Seller



Given the "mirror image quality" that results from the "zero-sum game" nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 4: Put Profit to Buyer



The breakeven price for a put position upon expiration is the exercise price minus the premium paid, $X - P_0$.

Underlying Assets

Exchange-traded options trade on four primary assets: individual stocks, foreign currency, stock indices, and futures.

Stock options. Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock. For example, if the last trade on a call option occurred at $3.60, the option contract would cost $360. After issuance, stock option contracts are adjusted for stock splits but not cash dividends. The primary U.S. exchanges for stock options are the Chicago Board Options Exchange (CBOE), Boston Options Exchange, NYSE Euronext, and the International Securities Exchange.

Currency options. Investors holding currency options receive the right to buy or sell an amount of foreign currency based on a domestic currency amount. For calls, a currency option is going to pay off only if the actual exchange rate is above a specified exercise rate. For puts, a currency option is going to pay off only if the actual exchange rate is below a specified exercise rate. The majority of currency options are traded on the over-the-counter market, while the remainder are exchange traded. The NASDAQ OMX trades European-style options for several currencies. Note that the unit size for currency options is considerably larger than stock options (i.e., 1 million units for yen and 10,000 units for other currencies).

Index options. Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter markets and the exchange-traded markets. The payoff on an index call is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the contract multiplier (typically 100).

Example: Index options

Assume you own a call option on an index with an exercise price equal to 950. The multiplier for this contract is 100. **Compute** the payoff on this option assuming that the index is 956 at expiration.

Answer:

The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the exercise price), multiplied by the contract multiplier. An equal amount will be deducted from the account of the index call option writer. In this example, the expiration date payoff is (956 – 950) × $100 = $600.

Futures options. American-style, exchange-traded options are most often utilized for futures contracts. Typically, the futures option expiration date is set to a date shortly before the expiration date of the futures contract. The market value of the underlying asset for futures options is the value of the underlying futures contract. The payoff for call options is calculated as the futures price less the strike price, while the payoff for put options is calculated as the strike price less the futures price.

Stock Options Specifications

LO 40.2: Explain the specification of exchange-traded stock option contracts, including that of nonstandard products.

Expiration

Options can be either American or European style. As mentioned previously, American options can be exercised throughout the life of the option, while European options can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options. Exchange-traded stock options are typically American-style options. The expiration dates of these options dictate how the option is named. For example, a June put option on Intel means that the option expires in June. The actual day of expiration is the Saturday following the third Friday of the expiration month. Different expiration cycles dictate the actual expiration months of a stock option over a given year. **Long-term equity anticipation securities** (LEAPS®) are simply long-dated options with expirations greater than one year. All LEAPS have January expirations.

Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced around \$20 have increments of \$2.50, stocks that are priced around \$50 have increments of \$5.00, and so on. The strike price is usually denoted as X and the underlying stock as S.

Moneyness, Time Value, and Intrinsic Value

An *option class* refers to all options of the same type, whether calls or puts. An *option series* refers to an option class with the same expiration. For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset is greater (less) than the strike price, the option is said to be in the money. An option price (or premium) prior to expiration has two components: the time value and the intrinsic value. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price [i.e., intrinsic value of a call option = max(0, S – X) and intrinsic value of a put option = max(0, X – S)]. The *time value* is the difference between the option premium and the intrinsic value.

Nonstandard Products

Nonstandard option products include flexible exchange (FLEX) options, exchange-traded fund (ETF) options, weekly options, binary options, credit event binary options (CEBOs), and deep out-of-the-money (DOOM) options.

FLEX options. FLEX options are exchange-traded options on equity indices and equities that allow some alteration of the options contract specifications. The nonstandard terms include alteration of the strike price, different expiration dates, or European-style (rather than the standard American-style). FLEX options were developed in order for the exchanges to better compete with the nonstandard options that trade over the counter. The minimum size for FLEX trades is typically 100 contracts.

ETF options. While similar to index options, ETF options are typically American-style options and utilize delivery of shares rather than cash at settlement.

Weekly options. *Weeklys* are short-term options that are created on a Thursday and have an expiration date on the Friday of the next week.

Binary options. Binary options generate discontinuous payoff profiles because they pay only one price ($100) at expiration if the asset value is above the strike price. The term binary means the option payoff has one of two states: the option pays $100 at expiration if the option is above the strike price or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

CEBOs. A CEBO is a specific form of credit default swap. The payoff in a CEBO is triggered if the reference entity suffers a qualifying credit event (e.g., bankruptcy, missed debt payment, or debt restructuring) prior to the option's expiration date (which always occurs in December). Option payoff, if any, occurs on the expiration date. CEBOs are European options that are cash settled.

DOOM options. These put options are structured to only be in the money in the event of a large downward price movement in the underlying asset. Due to their structure, the strike price of these options is quite low. In terms of protection, DOOM options are similar to credit default swaps. Note that this option type is always structured as a put option.

The Effect of Dividends and Stock Splits

In general, options are not adjusted for cash dividends. This will have option pricing consequences that will need to be incorporated into a valuation model. Options are adjusted for *stock splits*. For example, if a stock has a 2-for-1 stock split, then the strike price will be reduced by one-half and the number of shares underlying the option will double. In general, if a stock experiences a b-for-a stock split, the strike price becomes (a/b) of its previous value and the number of shares underlying the option is increased by multiples of (b/a). Stock dividends are dealt with in the same manner. For example, if a stock pays a 25% stock dividend, this is treated in the same manner as a 5-for-4 stock split.

Position and Exercise Limits

The number of options a trader can have on one stock is limited by the exchange. This is called a position limit. Additionally, short calls and long puts are considered to be part of the same position. The exercise limit equals the position limit and specifies the maximum number of option contracts that can be exercised by an individual over any five consecutive business days.

Option Trading

LO 40.3: Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options.

As mentioned, options are quoted relative to one underlying stock. To compute the actual option cost, the quote needs to be multiplied by 100. This is because an options contract represents an option on 100 shares of the underlying stock. The quotes will also include the strike, expiration month, volume, and the option class.

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option. If a trade is not an offsetting trade, then open interest increases by one contract.

Commissions

Option investors must consider the commission costs associated with their trading activity. Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount. The following example provides an illustration on how commission costs affect an option trade's profitability.

Example: Commission costs

An investor buys a call contract with a strike price of $260. The current price of the underlying stock is $245. Assume the option price is $10 and the contract is settled with shares rather than cash. Using the commission schedule for a discount broker below, **calculate** (1) the commission costs incurred by the investor based on the initial trade and (2) the investor's net profit if the stock price increases to $280 prior to expiration. Assume the cost to exercise the option is 1% of the trade amount and the cost to sell stock is also 1% of the trade amount.

Figure 5: Commission Schedule

Trade Amount	*Commission Rate*
≤ $3,000	$30 + 0.8% of trade amount
$3,001 to $14,999	$30 + 0.6% of trade amount
≥ $15,000	$30 + 0.4% of trade amount

Other details:
Minimum charge per contract: $4
Maximum charge per contract: $35

Answer:

1. Contract cost = $10 × 100 = $1,000

   Initial commission costs = $30 + ($1,000 × 0.8%) = $38. Because this exceeds the maximum contract charge, $35 is charged (i.e., the maximum contract charge).

2. Gross profit: $280 – $260 = $20 per share. $20 × 100 shares = $2,000

   Additional commission costs = 1% × 2 × $280 × 100 = $560

   Total commission costs = $35 + $560 = $595

   Net profit = $2,000 – $1,000 – $595 = $405

Due to the costs associated with exercising the option and then selling the stock, some retail investors may find it more efficient to simply sell the option to another investor.

One final note on option commission costs is that they fail to account for the cost embedded in the bid-offer spread. The cost associated with this spread for options can be calculated by multiplying the spread by 50%. For example, if the bid price is $12 and the offer price is $12.20, the associated cost for both the option buyer and option seller would be $0.10 per contract [(= $12.20 – $12.00) × 50%]. This cost is also present in stock transactions.

Margin Requirements

Options with maturities nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

Naked options (or *uncovered options*) refers to options in which the writer does not also own a position in the underlying asset. The size of the initial and maintenance margin for naked option writing is equal to the option premium plus a percentage of the underlying share price. Writing *covered calls* (selling a call option on a stock that is owned by the seller of the option) is far less risky than naked call writing.

The Options Clearing Corporation

Similar to a clearinghouse for futures, the **Options Clearing Corporation** (OCC) guarantees that buyers and sellers in the exchange-traded options market will honor their obligations and records all option positions. Exchange-traded options have no default risk because of the OCC, while over-the-counter options possess default risk. The OCC requires that all trades are cleared by one of its clearing members. OCC members must meet net capital requirements and help finance an emergency fund that is utilized in the event of a member default. Non-member brokers must contact a clearing member to clear their option trades. The OCC guarantees contract performance and therefore requires option writers to post margin as a means of supporting their obligation and option buyers to deposit required funds by the morning of the business day immediately following the day the option is purchased.

Exercising an Option

When an investor decides to exercise an option prior to contract expiration, her broker contacts the assigned OCC member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in the money after accounting for transaction costs will be exercised by brokers.

Other Option-Like Securities

Exchange-traded options are not issued by the company and delivery of shares associated with the exercise of exchange-traded options involves shares that are already outstanding. *Warrants* are often issued by a company to make a bond issue more attractive and will typically trade separately from the bond at some point. Warrants are like call options

except that, upon exercise, the company receives the strike price and may issue new shares to deliver. The same distinction applies to *employee stock options,* which are issued as an incentive to company employees and provide a benefit if the stock price rises above the exercise price. When an employee exercises incentive stock options, any shares issued by the company will increase the number of shares outstanding.

Convertible bonds contain a provision that gives the bondholder the option of exchanging the bond for a specified number of shares of the company's common stock. At exercise, the newly issued shares increase the number of shares outstanding and debt is retired based on the amount of bonds exchanged for the shares. There is a potential for dilution of the firm's common shares from newly issued shares with warrants, employee stock options, and convertible bonds that does not exist for exchange-traded options.

Key Concepts

LO 40.1

A call (put) option gives the owner the right to purchase (sell) the underlying asset at a strike price. When the owner executes this right, the option is said to be exercised. Because long (buy, purchase) option positions give the owner the right to exercise, the seller (short, writer) of the option has the obligation to meet the terms of the option.

American options may be exercised at any time up to and including the contract's expiration date, while European options can be exercised only on the contract's expiration date. Exchange-traded options are typically American options.

Primary types of exchange-traded options include option on individual stocks, foreign currency, stock indices, and futures.

LO 40.2

For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset price is greater (less) than the strike price, the option is said to be in the money. Options are not adjusted for cash dividends, but are adjusted for stock splits.

LEAPS are options with expiration dates greater than a year. Nonstandard option products include FLEX options, ETF options, weekly options, binary options, CEBOs, and DOOM options.

LO 40.3

Options with a maturity of nine months or fewer cannot be purchased on margin and must be paid in full due to the leverage effect of options. For options with longer maturities, investors can borrow up to 25% of the option value. Writers of options are required to have margin accounts with a broker.

Investors must account for commission costs when utilizing option. Commissions vary based on trade size and broker type. Commission rates typically are structured as a fixed dollar amount plus a percentage of the trade amount. In some instances, investors can earn higher profits by selling in-the-money options rather than exercising the options.

The Options Clearing Corporation (OCC) guarantees that buyers and sellers in the options market will honor their obligations and records all option positions. This minimizes default risk.

Warrants, employee stock options, and convertible bonds are option-like securities. Unlike options, these securities are issued by financial institutions or companies. The cost to the issuer of these securities is the possibility of increased dilution of the stock.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An investor owns a stock option that currently has a strike price of $100.

1. If the stock experiences a 4-to-1 split, the strike price becomes:
   A. $20.
   B. $25.
   C. $50.
   D. $100.

2. The number of shares now covered by each option contract is:
   A. 100.
   B. 200.
   C. 300.
   D. 400.

3. If an option is quoted at $2.75, the cost of one contract to the potential buyer is closest to:
   A. $0.275.
   B. $2.75.
   C. $275.00.
   D. $2,750.00.

4. Which of the following statements regarding option value or expiration is correct?
   I. American-style options are less valuable than European options.
   II. All options expire on the third Wednesday of the expiration month.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. Which of the following option characteristics is correct?
   I. A put option is in the money when the asset price is less than the strike price.
   II. LEAPS are long-term (over one-year) options that expire in December of each year.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. B $\frac{a}{b} = \frac{1}{4} \times \$100 = \$25$

2. D $\frac{b}{a} = \frac{4}{1} \times \$100 = \$400$ (Each option contract is originally for 100 shares.)

3. C Multiply the quote by 100 because each option contract is for 100 shares. $2.75 × 100 = $275.00

4. D American-style options are at least as valuable as European-style options. Options expire on the Saturday after the third Friday.

5. A A put option is in the money when the asset price is less than the strike price. LEAPS expire in January.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Properties of Stock Options

Topic 41

Exam Focus

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock; the strike price; the time to expiration; the volatility of the stock price; the risk-free rate; and dividends. The value of stock options have upper and lower pricing bounds. Be familiar with these pricing bounds as well as the relationships that exist between the value of European and American options.

Six Factors That Affect Option Prices

LO 41.1: Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options.

The following six factors will impact the value of an option:

1. S_0 = current stock price.
2. X = strike price of the option.
3. T = time to expiration of the option.
4. r = short-term risk-free interest rate over *T*.
5. D = present value of the dividend of the underlying stock.
6. σ = expected volatility of stock prices over *T*.

When evaluating a change in any one of the factors, hold the other factors constant.

Current Price of the Stock

For call options, as *S* increases (decreases), the value of the call increases (decreases). For put options, as *S* increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as *X* increases (decreases), the value of the call decreases (increases). For put options, as *X* increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a 1-month and 3-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and 3-month option price will fall when the dividend is paid in two months, the 1-month option may be worth more than the 3-month option.

The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Upper and Lower Pricing Bounds

LO 41.2: Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

In addition to those previously introduced, consider the following variables:

- c = value of a European call option.
- C = value of an American call option.
- p = value of a European put option.
- P = value of an American put option.
- S_T = value of the stock at expiration.

Also, assume in the following examples that there are no transaction costs, all profits are taxed at the same rate, and borrowing and lending can be done at the risk-free rate.

Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

Lower Pricing Bounds for European Calls on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_1: one European call, *c*, with exercise price *X* plus a zero-coupon risk-free bond that pays *X* at *T*.
- Portfolio P_2: one share of the underlying stock, *S*.

At expiration, *T*, Portfolio P_1 will always be the greater of *X* (when the option expires out-of-the-money) or S_T (when the option expires in-the-money). Portfolio P_2, on the other hand, will always be worth S_T. Therefore, P_1 is always worth at least as much as P_2 at

expiration. If we know that at T, $P_1 \geq P_2$, then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + Xe^{-rT} \geq S_0$$

Since the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a nondividend-paying stock is:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Lower Pricing Bounds for European Puts on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_3: one European put, p, plus one share of the underlying stock, S.
- Portfolio P_4: zero-coupon risk-free bond that pays X at T.

At expiration, T, Portfolio P_3 will always be the greater of X (when the option expires in-the-money) or S_T (when the option expires out-of-the-money). Portfolio P_4, on the other hand, will always be worth X. Therefore, P_3 is always worth at least as much as P_4 at expiration. If we know that at T, $P_3 \geq P_4$, it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq Xe^{-rT}$$

Since the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a nondividend-paying stock is:

$$p \geq \max(Xe^{-rT} - S_0, 0)$$

Computing Option Values Using Put-Call Parity

LO 41.3: Explain put-call parity and apply it to the valuation of European and American stock options.

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays X at maturity and a call with exercise price X. The payoff for a fiduciary call at expiration is X when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in the money, and S when the put is out of the money.

Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in the money, the call is out of the money, and both portfolios pay X at expiration.

Similarly, when the put is out of the money and the call is in the money, both portfolios pay S at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$\begin{aligned} S &= c - p + Xe^{-rT} \\ p &= c - S + Xe^{-rT} \\ c &= S + p - Xe^{-rT} \\ Xe^{-rT} &= S + p - c \end{aligned}$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + Xe^{-rT}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

Example: Call option valuation using put-call parity

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. Estimate the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - Xe^{-rT}$$

$$\text{call} = \$1.50 + \$52 - \$50e^{-0.0125} = \$4.12$$

This means that if a 3-month, \$50 call is available, it should be priced at \$4.12 per share.

Lower Pricing Bounds for an American Call Option on a Nondividend-Paying Stock

LO 41.4: Explain the early exercise features of American call and put options.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Since the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max(S_0 - Xe^{-rT}, 0)$$

Note that when an American call is exercised, it is only worth $S_0 - X$. Since this value is never larger than $S_0 - Xe^{-rT}$ for any r and $T > 0$, it is never optimal to exercise early. In other words, the investor can keep the cash equal to X, which would be used to exercise the option early, and invest that cash to earn interest until expiration. Since exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a nondividend-paying stock before the expiration date (i.e., $c = C$).

Lower Pricing Bounds for an American Put Option on a Nondividend-Paying Stock

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff ($X - S_0$) can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, Xe^{rT}, is always worth more than a later exercise, X. We know that:

$P \geq p \geq \max(Xe^{-rT} - S_0, 0)$ for the same reasons that $C \geq c$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$$P \geq \max(X - S_0, 0)$$

Figure 2 summarizes what we now know regarding the boundary prices for American and European options.

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Professor's Note: For the exam, know the price limits in Figure 2. You will not be asked to derive them, but you may be expected to use them.

Example: Minimum prices for American vs. European puts

Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

Answer:

$$P \geq \max(0, X - S_0) = \max(0, 2) = \$2$$

$$p \geq \max(0, Xe^{-rT} - S_0) = \max(0, 65e^{-0.0167} - 63) = \$0.92$$

Example: Minimum prices for American vs. European calls

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

Answer:

$$C \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

$$c \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

Example: American put option bounds

Consider an American call and put option on stock XYZ. Both options have the same 1-year expiration and a strike price of \$20. The stock is currently priced at \$22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at \$4?

Answer:

The upper and lower bounds on the difference between the American call and American put options are:

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

$$S_0 - X = 22 - 20 = \$2$$

$$S_0 - Xe^{-rT} = 22 - 20e^{-0.06(1)} = 22 - 18.84 = \$3.16$$

$$\$2 \leq C - P \leq \$3.16$$

or

$$-\$2 \geq P - C \geq -\$3.16$$

Therefore, when the American call is valued at \$4, the upper and lower bounds on the American put option will be:

$$\$2 \geq P \geq \$0.84$$

The Impact of Dividends on Option Pricing Bounds

Since most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio P_6: one European call option, *c*, plus cash equal to $D + Xe^{-rT}$.
- Portfolio P_7: one share of the underlying stock, *S*.

Similar to the development of the $c \geq \max(S_0 - Xe^{-rT}, 0)$ equation, Portfolio P_6 is always at least as large as P_7, or:

$$c \geq S_0 - D - Xe^{-rT}$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio P_8: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_9: cash equal to $D + Xe^{-rT}$.

Using the same development as the $p \geq \max(Xe^{-rT} - S_0, 0)$ equation:

$$p \geq D + Xe^{-rT} - S_0$$

All else equal, the payment of a dividend will increase the lower pricing bound for a put option.

Impact of Dividends on Early Exercise for American Calls and Put-Call Parity

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + Xe^{-rT}$$

This equation is verified using the same development as was used to derive the $p + S_0 = c + Xe^{-rT}$ equation. The $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

Key Concepts

LO 41.1

Six factors influence the value of an option: current value of the underlying asset (stock); the strike price; the time to expiration of the option; the volatility of the stock price; the risk-free rate; and dividends.

With the exception of time to expiration, all of these factors affect European- and American-style options in the same way.

LO 41.2

Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.

When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.

LO 41.3

Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

$$p + S_0 = c + Xe^{-rT}$$

LO 41.4

It is never optimal to exercise an American call option on nondividend-paying stock prior to expiration.

American put options on nondividend-paying stocks can be optimally exercised prior to expiration if the put is sufficiently in-the-money.

Call options are always worth more than corresponding put options prior to expiration when both are at-the-money.

The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and strike price, and above by the difference between the current stock price and the present value of the strike price.

Concept Checkers

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
   A. XYZ declares a 3-for-1 stock split.
   B. XYZ raises its quarterly dividend from \$0.15 per share to \$0.17 per share.
   C. The Federal Reserve lowers interest rates by 0.25% in an effort to stimulate the economy.
   D. Investors believe the volatility of XYZ stock has declined.

2. Consider a European put option on a stock trading at \$50. The put option has an expiration of six months, a strike price of \$40, and a risk-free rate of 5%. The lower bound and upper bound on the put are:
   A. \$10, \$40.00.
   B. \$10, \$39.01.
   C. \$0, \$40.00.
   D. \$0, \$39.01.

3. Consider a 1-year European put option that is currently valued at \$5 on a \$25 stock and a strike of \$27.50. The 1-year risk-free rate is 6%. Which of the following is closest to the value of the corresponding call option?
   A. \$0.00.
   B. \$3.89.
   C. \$4.10.
   D. \$5.00.

4. Consider an American call and put option on the same stock. Both options have the same 1-year expiration and a strike price of \$45. The stock is currently priced at \$50, and the annual interest rate is 10%. Which of the following could be the difference in the two option values?
   A. \$4.95.
   B. \$7.95.
   C. \$9.35.
   D. \$12.50.

5. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
   A. buying a call, buying ABC stock, and buying a zero-coupon bond.
   B. buying a call, selling ABC stock, and buying a zero-coupon bond.
   C. selling a call, selling ABC stock, and buying a zero-coupon bond.
   D. buying a call, selling ABC stock, and selling a zero-coupon bond.

Concept Checker Answers

1. **A** After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and lower volatility of the price of the underlying stock, will all decrease the value of a European call option.

2. **D** The upper bound is the present value of the exercise price: $\$40 \times e^{-0.05 \times 0.5} = \$39.01$. Since the put is out-of-the-money, the lower bound is zero.

3. **C** $c = p - Xe^{-rT} + S_0 = \$5 - \$27.50e^{-0.06 \times 1} + \$25 = \$4.10$

4. **B** The upper and lower bounds are: $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ or $\$5 \leq C - P \leq \$9.28$. Only \$7.95 falls within the bounds.

5. **B** The formula for put-call parity is $p + S_0 = c + Xe^{-rT}$. Rearranging to solve for the price of a put, we have $p = c - S_0 + Xe^{-rT}$.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Trading Strategies Involving Options

Topic 42

Exam Focus

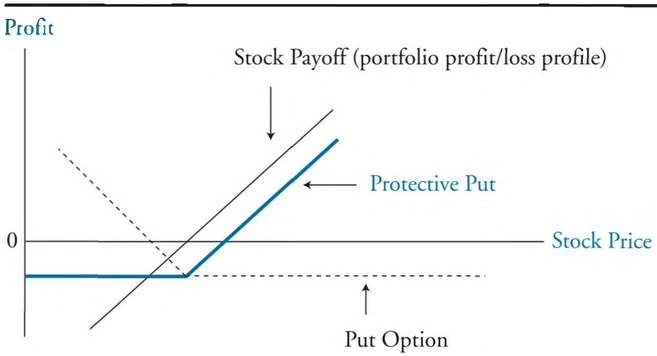
Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This topic describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate the payoff for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

Covered Calls and Protective Puts

LO 42.1: Explain the motivation to initiate a covered call or a protective put strategy.

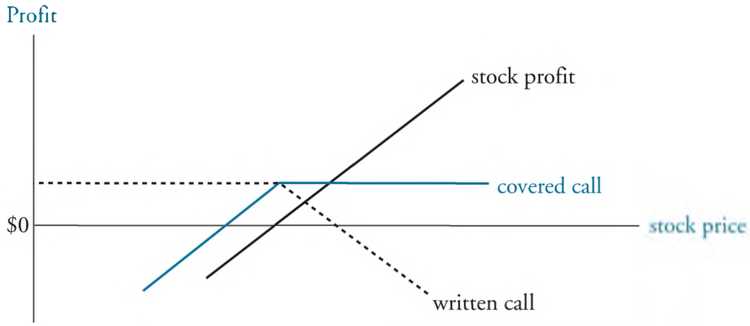
When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, P_0. You will see by the diagram in Figure 1 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put, P_0. Notice that the combined strategy looks very much like a call option. This should not be surprising since put-call parity requires that $p + S_0$ be the same as $c + Xe^{-rT}$. Figure 1 illustrates this property.

Figure 1: Protective Put Strategy



Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 2: Profit Profile for a Covered Call



Spread Strategies

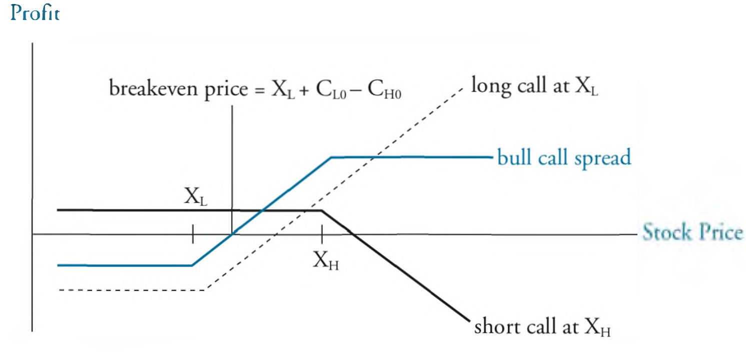
LO 42.2: Describe the use and calculate the payoffs of various spread strategies.

Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

Bull and Bear Spreads

In a *bull call spread,* the buyer of the spread purchases a call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a call with a higher exercise price, X_H. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

Figure 3: Bull Call Spread



Example: Bull call spread

An investor purchases a call for C_{L0} = \$3.00 with a strike of X = \$40 and sells a call for C_{H0} = \$1.00 with a strike price of \$50. **Compute** the payoff of a bull call spread strategy when the price of the stock is at \$45.

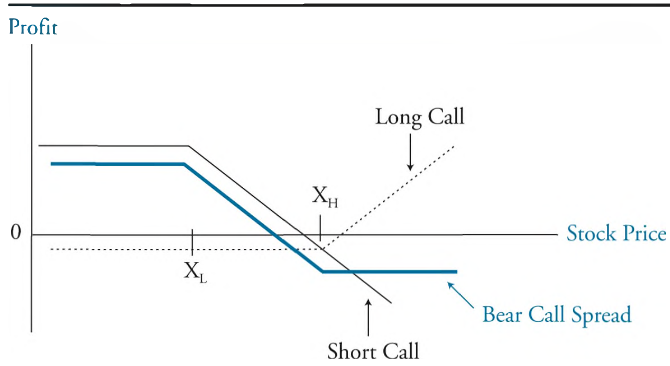
Answer:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 4.

Figure 4: Bear Call Spread



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread* the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

Example: Bear put spread

An investor sells a put for P_{L0} = \$3.00 with a strike of X = \$20 and purchases a put for P_{H0} = \$4.50 with a strike price of \$40. **Compute** the payoff of a bear put spread strategy when the price of the stock is at \$35.

Answer:

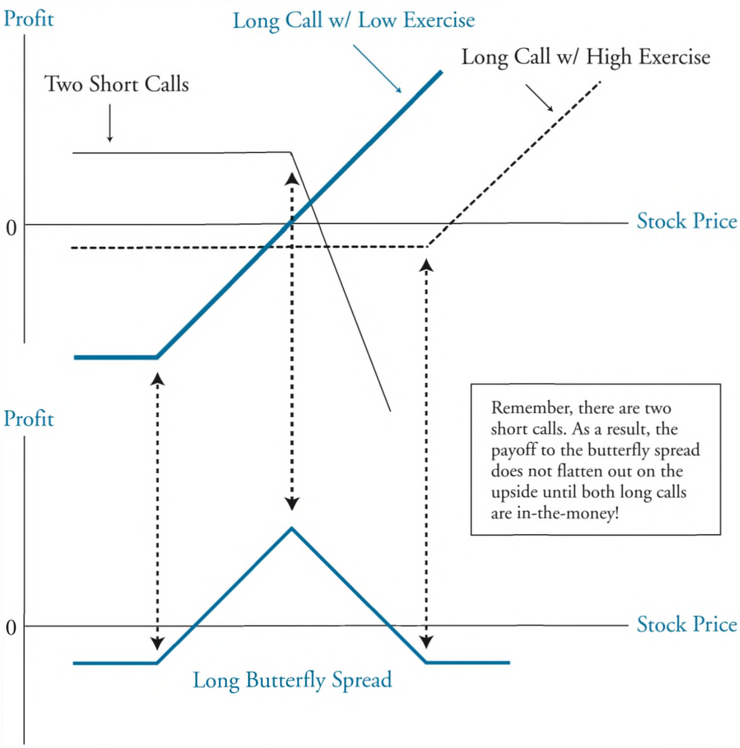
$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

Butterfly Spreads

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 5 illustrate the construction and payoffs of a butterfly spread.

Figure 5: Butterfly Spread Construction and Behavior



Example: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by C_{L0} = \$ 7.00 and X_L = \$55.
- Buys one call defined by C_{H0} = \$2.00 and X_H = \$65.
- Sell two calls defined by C_{M0} = \$4.00 and X_M = \$60.

Compute the payoff of a butterfly spread strategy with calls when the stock is at \$60.

Answer:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

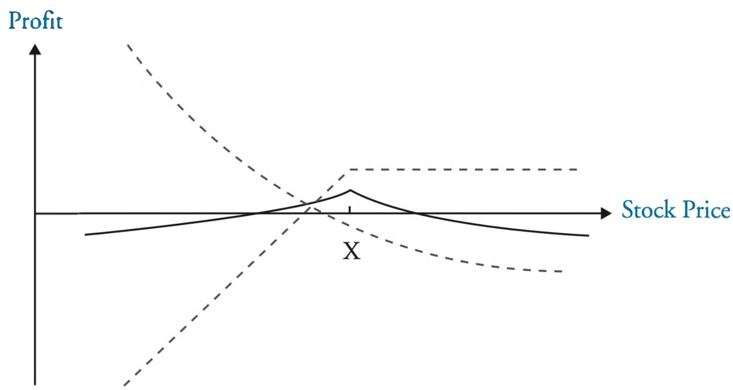
$$\text{profit} = \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 + 2(4) - 2 = \$4.00$$

To create a butterfly spread with put options, the investor would buy a low and high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 6 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits only if the stock remains in a narrow range, but losses are limited. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

Figure 6: Calendar Spread (Using Two Put Options)



Calendar spreads are categorized differently depending on the relationship between the strike price and the current stock price. The strategy is referred to as a **neutral calendar spread** if the strike price is close to the current stock price. A **bullish calendar spread** has a strike price above the current stock price, and a **bearish calendar spread** has a strike price below the current stock price.

A **reverse calendar spread** produces a payoff that is opposite of the graph shown in Figure 6. Instead of selling a short-dated option and buying a long-dated option, the investor of a reverse calendar spread will buy a short-dated option and sell a long-dated option. The investor will profit when the stock is well above or below the strike price and will suffer a loss if the stock is near the strike price.

Diagonal Spreads

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

Box Spreads

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price (X_H) minus the low exercise price (X_L). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

When the profit from this strategy is different than zero, an investor can capitalize on the arbitrage opportunity by either buying or selling the box. If the profit is positive, the investor will create a long box spread by buying a call at X_L, selling a call at X_H, buying a put at X_H, and selling a put at X_L. If the profit is negative, the investor will create a short box spread by buying a call at X_H, selling a call at X_L, buying a put at X_L, and selling a put at X_H. Note that box spread arbitrage is only successful with European options.

Combination Strategies

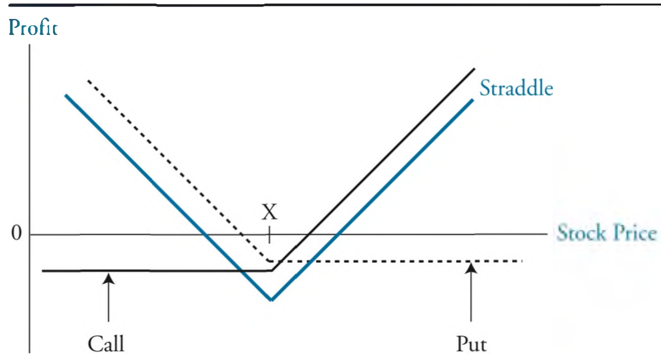
LO 42.3: Describe the use and explain the payoff functions of combination strategies.

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

Straddle

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Figure 7 illustrates the payoff for a long straddle position. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

Figure 7: Long Straddle Profit/Loss



Example: Straddle

An investor purchases a call on a stock, with an exercise price of \$45 and a premium of \$3, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. **Compute** the payoff of a straddle strategy if the stock is at \$35.

Answer:

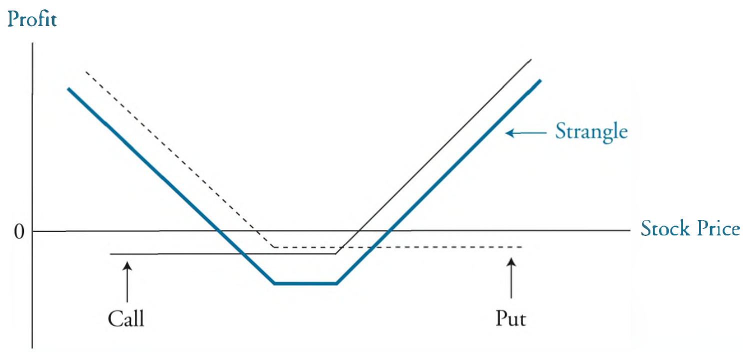
$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

Strangle

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 8. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

Figure 8: Long Strangle Profit/Loss



Example: Strangle

An investor purchases a call on a stock, with an exercise price of \$50 and a premium of \$1.50, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. **Compute** the payoff of a strangle strategy if the stock is at \$40.

Answer:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

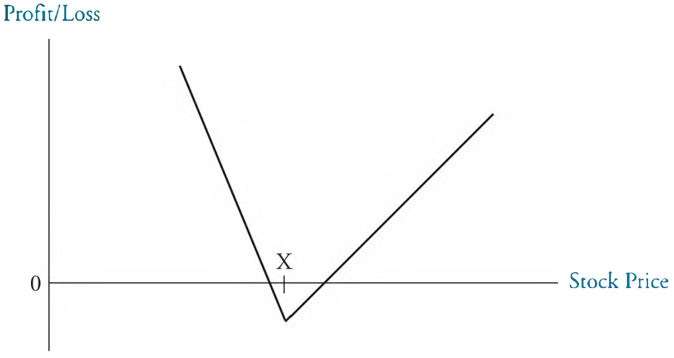
$$\text{profit} = \max(0, 40 - \$50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

A short strangle (or a top vertical combination) is similar to the short straddle.

Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Figure 9 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

Figure 9: Strip Profit/Loss

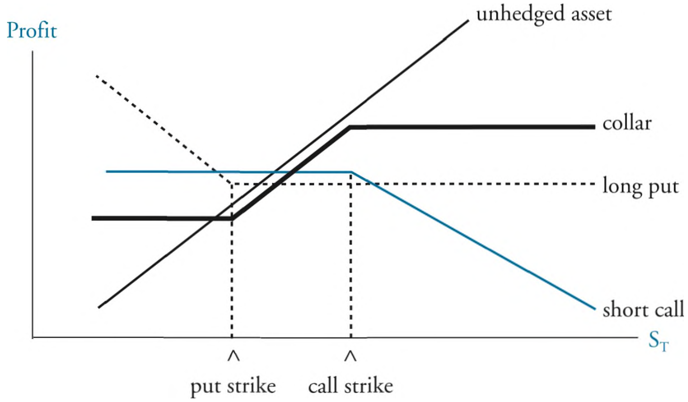


A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

Collar

A *collar* is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar.**

Figure 10: Collar



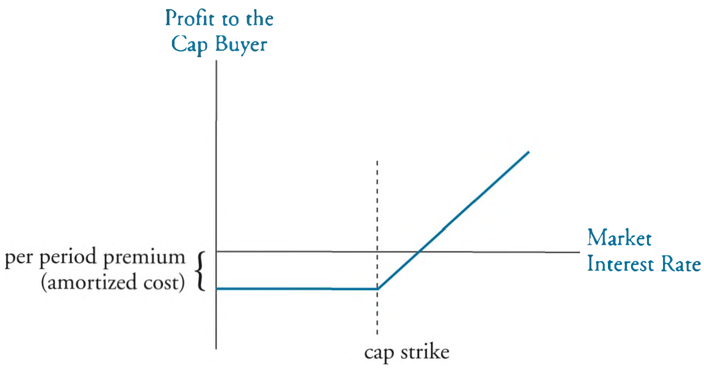
Interest Rate Caps and Floors

An **interest rate cap** is an agreement in which one party agrees to pay the other at regular intervals over a certain period of time when the benchmark interest rate (e.g., LIBOR) exceeds the strike rate specified in the contract. This strike rate is called the **cap rate**. For example, the seller of a cap might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is greater than a cap rate of 6%.

The buyer of a cap has a position similar to that of a buyer of a call on LIBOR, both of whom benefit when interest rates rise. Because an interest rate cap is a multi-period agreement, a cap is actually a portfolio of call options on LIBOR called **caplets**. For example, the 2-year cap discussed above is actually a portfolio of eight interest rate options with different maturity dates.

The cap buyer pays a premium to the seller and exercises the cap if the market rate of interest rises above the cap strike. The diagram in Figure 11 illustrates the profits of an interest rate cap at the end of one particular settlement period. It has the familiar shape of a long position in a call option.

Figure 11: Profit to a Long Cap

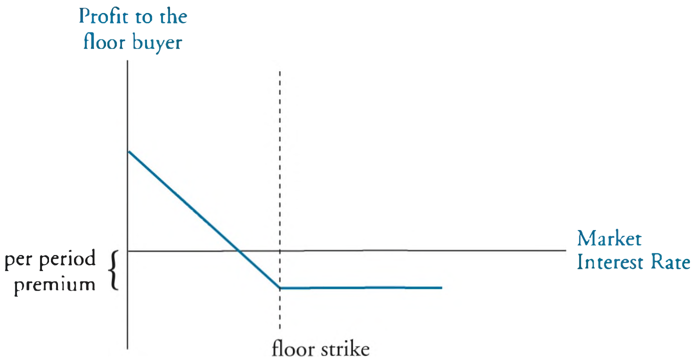


An **interest rate floor** is an agreement in which one party agrees to pay the other at regular intervals over a certain time period when the benchmark interest rate (e.g., LIBOR) falls below the strike rate specified in the contract. This strike rate is called the **floor rate**. For example, the seller of a floor might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is less than a floor rate of 4%.

The buyer of a floor benefits from an interest rate decrease and, therefore, has a position that is similar to that of a buyer of a put on LIBOR, who benefits when interest rates fall and the price of the instrument rises. Once again, because a floor is a multi-period agreement, a floor is actually a portfolio of put options on LIBOR called **floorlets**.

The floor buyer pays a premium and exercises the floor if the market rate of interest falls below the floor strike. The diagram in Figure 12 illustrates the profits of an interest rate floor at the end of one particular settlement period. It has the same shape as a long put option.

Figure 12: Profit to a Long Floor



Options are traded both on *interest rates* and on *prices* of fixed-income securities. So far we've talked about options on interest rates. The values of comparable options on rates and prices respond differently to changes in interest rates because of the inverse relationship

between bond yields and bond prices. Figure 13 outlines how each type of option responds to changes in yields and bond prices.

Figure 13: Options on Rate vs. Options on Prices

Option	*If Rates* Increase *and Bond Prices* Decrease	*If Rates* Decrease *and Bond Prices* Increase
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

We can also interpret caps and floors in terms of options on the prices of fixed-income securities:

- A long cap is equivalent to a portfolio of long put options on fixed-income security prices.
- A long floor is equivalent to a portfolio of long call options on fixed-income security prices.

An **interest rate collar** is a simultaneous position in a floor and a cap on the same benchmark rate over the same period with the same settlement dates. There are two types of collars:

- The first type of collar is to purchase a cap and sell a floor. For example, an investor with a LIBOR-based liability could purchase a cap on LIBOR at 8% and simultaneously sell a floor on LIBOR at 4% over the next year. The investor has now hedged the liability so that the borrowing costs will stay within the "collar" of 4% to 8%. If the cap and floor rates are set so that the premium paid from buying the cap is exactly offset by the premium received from selling the floor, the collar is called a "zero-cost" collar.
- The second type of collar is to purchase a floor and sell a cap. For example, an investor with a LIBOR-based asset could purchase a floor on LIBOR at 3% and simultaneously sell a cap at 7% over the next year. The investor has now hedged the asset so the returns will stay within the collar of 3% to 7%. The investor can create a zero-cost collar by choosing the cap and floor rates so that the premium paid on the floor offsets the premium received on the cap.

Key Concepts

LO 42.1

Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.

LO 42.2

Spread strategies combine options in the same option class to generate various payoff profiles.

The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost.

A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.

A calendar spread is created by transacting in two options that have the same strike price but different expirations.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.

In a diagonal spread, options can have different strike prices and different expirations.

Bull call spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

Bear put spread payoff:

$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

Butterfly spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

LO 42.3

Combination strategies combine puts and calls to generate various payoff strategies.

A long straddle (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.

A strangle (or bottom vertical combination) is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.

A strip is betting on volatility but is more bearish since it pays off more on the down side.

A strap is betting on volatility but is more bullish since it pays off more on the up side.

Straddle payoff:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

Strangle payoff:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

Concept Checkers

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?

   I. Short butterfly spread.
   II. Bearish calendar spread.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following will create a bear spread?
   A. Buy a call with a strike price of X = 45 and sell a call with a strike price of X = 50.
   B. Buy a call with a strike price of X = 50 and buy a put with a strike price of X = 55.
   C. Buy a put with a strike price of X = 45 and sell a put with a strike price of X = 50.
   D. Buy a call with a strike price of X = 50 and sell a call with a strike price of X = 45.

3. An investor believes that a stock will either increase or decrease greatly in value over the next few months, but believes a down move is more likely. Which of the following strategies will be the best for this investor?
   A. A protective put.
   B. An at-the-money strip.
   C. An at-the-money strap.
   D. A top vertical combination.

4. An investor constructs a long straddle by buying an April $30 call for $4 and buying an April $30 put for $3. If the price of the underlying shares is $27 at expiration, what is the profit on the position?
   A. –$4.
   B. –$2.
   C. $2.
   D. $3.

5. Consider an option strategy where an investor buys one call option with an exercise price of $55 for $7, sells two call options with an exercise price of $60 for $4, and buys one call option with an exercise price of $65 for $2. If the stock price declines to $25, what will be the profit or loss on the strategy?
   A. –$3.
   B. –$1.
   C. $1.
   D. $2.

Concept Checker Answers

1. **A** A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates.

2. **D** Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid.

3. **B** An at-the-money strip bets on volatility but is more bearish since it pays off more on the downside.

4. **A** The sum of the premiums paid for the position is \$7. With the underlying stock at \$27, the put will be worth \$3, while the call option will be worthless. The value of the position is $(-\$7 + \$3) = -\$4$.

5. **B** The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sell two calls with a price in between. In this case, if the option moves to \$25, none of the call options will be in the money, so the profit is equal to the net premium paid, which is $-\$7 + (2 \times \$4) - \$2 = -\$1$.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Exotic Options

Topic 43

Exam Focus

In this topic, we define and discuss the important characteristics of a variety of exotic options. The difference between exotic options and more traditional exchange-traded instruments is also highlighted. Be familiar with the payoff structures for the various exotic options discussed.

Evaluating Exotic Options

LO 43.1: Define and contrast exotic derivatives and plain vanilla derivatives.

LO 43.2: Describe some of the factors that drive the development of exotic products.

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other over-the-counter (OTC) derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need for hedging that cannot be met by plain vanilla derivatives. With plain vanilla derivatives, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic derivatives are developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

Four questions that should be considered when evaluating exotic derivative strategies are:

- Will the strategy pay in the right circumstances to provide an effective hedge? Problems with understanding the payoff of the exotic derivative and credit risk of the derivative strategy can lead to a difference between the payoff the user expects and the actual payoff received.
- What is the cost of the exotic derivative hedging strategy?
- Is a pricing model needed, and does the user have the appropriate pricing model to estimate dealer cost and monitor the value of non-traded derivatives over time?
- How is a derivative position reversed? Note that the costs of exiting a position or strategy may involve penalties and large bid-ask spreads or require a pricing model to evaluate alternatives.

Using Packages to Formulate a Zero-Cost Product

LO 43.3: Explain how any derivative can be converted into a zero-cost product.

A package is defined as some combination of standard European options, forwards, cash, and the underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

For example, consider a zero-cost short collar. A short collar combines a long standard put option with an exercise price X_L and a short standard call option with exercise price X_H (where $X_L < X_H$). If the premium the investor pays for the put option is exactly offset by the premium the investor receives for the short call position, the investor's net cost for implementing the short collar strategy is zero. In any case where the investor's cash outflows from long positions are offset by cash inflows from short positions, the investor can use a package to create a zero-cost product.

Transforming Standard American Options Into Nonstandard American Options

LO 43.4: Describe how standard American options can be transformed into nonstandard American options.

Recall that standard exchange-traded American options can be exercised at any time prior to expiration. If some of the available expiration periods are restricted, or changes are made to other standard features, standard options become what we refer to as **nonstandard options**. Nonstandard options are common in the over-the-counter (OTC) market.

There are three common features that transform standard American options into nonstandard options:

- The most common transformation can be made to restrict early exercise to certain dates (e.g., a three month call option may only be exercised on the last day of each month.) This type of transformation results in a **Bermudan option.**
- Early exercise can be limited to a certain portion of the life of the option (e.g., there is a "lock out" period that does not allow a 6-month call option to be exercised in the first three months of the call's life).
- The option's strike price may change (e.g., the strike price of a 3-year call option with a strike price of 40 at initiation may rise to 44 in year 2 and 48 in year 3).

Exotic Option Payoff Structures

LO 43.5: Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options.

Gap Options

A gap option has two strike prices, X_1 and X_2. (X_2 is sometimes referred to as the trigger price.) If these two strike prices are equal, the gap option payoff will be the same as an ordinary option. If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices. Gap options can be valued with a slight modification to the Black-Scholes-Merton option pricing model, which will be discussed in Book 4.

For a *gap call option*, if X_2 is greater than X_1, and the stock price at maturity, S_T, is greater than the trigger price, X_2, then the payoff for the call option will be equal to $S_T - X_1$. If the stock price is less than or equal to X_2, the payoff will be zero. Note that a negative payoff can occur if the stock price is greater than X_2 and X_2 is less than X_1. In this case, the payoff will be reduced by $X_2 - X_1$.

For a *gap put option*, if X_2 is less than X_1, and the stock price at maturity, S_T, is less than the trigger price, X_2, then the payoff for the put option will be equal to $X_1 - S_T$. If the stock price is greater than or equal to X_2, the payoff will be zero. A negative payoff can occur if the stock price is less than X_2 and X_2 is greater than X_1. Like with a gap call option, if this is the case, the payoff will be reduced by $X_2 - X_1$.

Forward Start Options

Forward start options are options that begin their existence at some time in the future. For example, today an investor may purchase a 3-month call option that will not come into existence until six months from today. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed. Note that when the underlying asset is a nondividend paying stock, the value of a forward start option will be identical to the value of a European at-the-money option with the same time to expiration as the forward start option.

Compound Options

Compound options are options on options. There are four key types of compound options:

- A *call on a call* gives the investor the right to buy a call option at a set price for a set period of time.
- A *call on a put* gives the investor the right to buy a put option at a set price for a set period of time.

- A *put on a call* gives the investor the right to sell a call option at a set price for a set period of time.
- A *put on a put* gives the investor the right to sell a put option at a set price for a set period of time.

Compound options have two levels of the underlying that determine their value—the value of the underlying option, which in turn is determined by the value of the underlying asset.

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

Chooser Options

This interesting option allows the owner, after a certain period of time has elapsed, to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

Barrier Options

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options, and essentially come in either *knock-out* or *knock-in* flavors. Specific types of barrier options are:

- *Down-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Down-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Up-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits a barrier level, which is set above the current stock value.
- *Up-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the above-current stock-price barrier level.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire.

Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term binary means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

In the case of a **cash-or-nothing call**, a fixed amount, Q, is paid if the asset ends up above the strike price. Since the Black-Scholes-Merton formula denotes $N(d_2)$ as the probability of the asset price being above the strike price, the value of a cash-or-nothing call is equal to $Qe^{-rT}N(d_2)$.

An **asset-or-nothing call** pays the value of the stock when the contract is initiated if the stock price ends up above the strike price at expiration. The corresponding value for this option is $S_0e^{-qT}N(d_1)$, where q is the continuous dividend yield.

Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating lookback put** pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Lookback options can also be fixed when an exercise price is specified. A **fixed lookback call** has a payoff function that is identical to a European call option. However, for this exotic option, the final stock price (or expiration price) in the European call option payoff is replaced by the maximum price during the option's life. Similarly, a **fixed lookback put** has a payoff like a European put option but replaces the final stock price with the minimum price during the option's life.

Shout Options

A shout option allows the owner to pick a date when he "shouts" to the option seller, which then translates into an intrinsic value of the option at the time of the shout. At option expiration, the owner receives the maximum of the shout intrinsic value or the option expiration intrinsic value. In other words, for a shout call option, even if the price of the stock falls after the shout, the investor has locked in the difference between the price of the stock and the shout price. If the stock continues to rise, the shout option will have a payoff consistent with a standard call option. Note that most shout options allow for one "shout" during the option's life.

Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation. If the average price or strike price for an Asian option is based on a geometric average, then using an option pricing model is not a problem because a geometric average is lognormal. However, most Asian options base their average calculations on arithmetic averages, which complicates the pricing process. In this case, a lognormal distribution of prices is assumed, which provides an adequate approximation.

Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable to the U.S. investor than yen. Other applications, such as tender offers to exchange one stock for another, also arise in certain situations.

Basket Options

Basket options are simply options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as **rainbow options**.

Volatility and Variance Swaps

LO 43.6: Describe and contrast volatility and variance swaps.

A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a pre-specified fixed volatility while the other side pays based on realized volatility. Unlike the exotic options we have discussed thus far, volatility swaps are a bet on volatility alone as opposed to a bet on volatility and the price of the underlying asset.

Much like a volatility swap, a **variance swap** involves exchanging a pre-specified fixed variance rate for a realized variance rate. The variance rate being exchanged is simply the square of the volatility rate. However, unlike volatility swaps, variance swaps are easier to price and hedge since they can be replicated using a collection of call and put options.

Issues in Hedging Exotic Options

LO 43.7: Explain the basic premise of static option replication and how it can be applied to hedging exotic options.

The typical dynamic option-hedging situation uses option Greeks to measure sensitivity of the option value to changes in underlying asset characteristics (i.e., creating a delta-neutral portfolio). Hedging is simpler with some exotic options than it is with plain vanilla options. Asian options, for instance, depend on the average price of the underlying. Through time, the uncertainty of the average value gets smaller. Hence, the option begins to become less sensitive to changes in the value of the security because the payoff can be estimated more accurately.

Hedging positions in barrier and other exotic options are not so straightforward. This type of hedging requires the replication of a portfolio that is exactly opposite to the option position. When the replication portfolio requires frequent adjustments to the holdings in the underlying assets, the hedging procedure is referred to as dynamic options replication. **Dynamic options replication** requires frequent trading, which makes it costly to implement.

As an alternative, a **static options replication** approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once, which drastically reduces the transaction costs associated with dynamic rebalancing.

Key Concepts

LO 43.1

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other OTC derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need.

LO 43.2

The main purpose for the development of exotic derivatives is to provide a unique hedge for a firm's underlying assets. Additional reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

LO 43.3

Packages are portfolios of European options, forwards, cash, and the underlying asset. Given that packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

LO 43.4

Restricting exercise dates and changing strike prices can transform standard options into nonstandard options.

LO 43.5

A gap option has two strike prices. If the two strike prices differ and the payoff is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward start options are options that commence in the future.

A compound option is defined as an option on another option.

Chooser options allow the owner to choose whether the option is a call or a put, after option initiation.

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.

Binary options either pay nothing (if price is below strike price) or a fixed amount at expiration.

Lookback options depend on the maximum or minimum value of the underlying asset during the life of the option.

Shout options allow the owner to receive either the intrinsic value of the option at the shout date or at expiration, whichever is greater.

Asian options have payoff profiles that depend on the average underlying asset price over the life of the option.

An exchange option is an option to exchange one asset for another.

Basket options allow the owner to buy or sell portfolios of assets. Exotic options that involve several different assets are more generally referred to as rainbow options.

LO 43.6

A volatility swap involves the exchange of volatility based on a notional principal. A variance swap involves exchanging a pre-specified fixed variance rate for a realized variance rate.

LO 43.7

Exotic options can be hedged in either a dynamic or static context, depending on the characteristics of the option.

Concept Checkers

1. A down-and-in call option is an option that comes into existence only when the underlying asset price:
   A. rises to a set barrier level.
   B. falls to a set barrier level.
   C. falls to a set average barrier level.
   D. rises to a set average barrier level.

2. A cash-or-nothing put option has a payout profile equivalent to zero or:
   A. the underlying asset price if the value of the asset ends below the strike price.
   B. the underlying asset price if the value of the asset ends above the strike price.
   C. a set amount if the value of the asset ends below the strike price.
   D. a set amount if the value of the asset ends above the strike price.

3. An Asian option can be hedged dynamically because the:
   A. average value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   B. average value of the underlying asset price increases uncertainty the closer the option gets to expiration.
   C. maximum value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
   D. minimum value of the underlying asset price increases uncertainty the closer the option gets to expiration.

4. Which of the following options is most likely to have a negative vega?
   A. A chooser option close to expiration.
   B. A forward start put option before the start date.
   C. An Asian put option close to the beginning of the option's life.
   D. An up and out put when the stock price is close to the barrier.

5. Under which of the following circumstances would the value of an up and out call option be zero?
   A. The strike price is above the barrier price.
   B. The stock price is below the barrier price.
   C. The stock price is above the strike price.
   D. The stock price is below the strike price.

Concept Checker Answers

1. **B** Down-and-in call options are standard options that come into existence only if the asset price falls to a set barrier price level, which is set below the current stock price.

2. **C** Cash-or-nothing put options pay only a set amount if the stock price ends below the strike price. These options differ from standard put options because the payment is a set amount that does not continuously increase with the decrease in stock price.

3. **A** Dynamic hedging can be used to hedge Asian options because uncertainty in the expiration value is decreased the closer one gets to expiration. This occurs because the intrinsic value becomes "set" due to the averaging effect over the life of the option.

4. **D** Vega is the sensitivity of the price of an option to changes in volatility of the underlying stock. For most options, vega is always positive—as volatility of the underlying stock increases, the price of the option also increases. An exception would be a knockout barrier option when the stock price is close to the barrier. Higher volatility means the barrier is more likely to be reached and the option will cease to exist.

5. **A** With an up and out call, if the stock price rises beyond the barrier price, the option ceases to exist. It therefore follows that if the strike price is above the barrier price, the option will never come into the money because the option will cease to exist before the option will ever come into the money.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Commodity Forwards and Futures

Topic 44

Exam Focus

This topic on commodity forwards and futures focuses on the pricing relationships that exist when commodities have characteristics such as lease rates, storage costs, and/or convenience yields. Before you begin this topic, recall the no-arbitrage pricing relationships for futures contracts that were discussed in Topic 37 (Determination of Forward and Futures Prices). You should understand the basic futures pricing equation and how it is adjusted for lease rates, storage costs, and/or convenience yields.

Pricing Commodity Forwards and Futures

LO 44.1: Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.

LO 44.2: Explain the basic equilibrium formula for pricing commodity forwards.

LO 44.13: Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price.

Professor's Note: LO 44.1 is addressed throughout this topic.

Commodity and financial forward contracts are similar in some regards. For example, the prices of both are logically based upon expected spot prices. Some financial forwards (e.g., S&P 500 Index) are based upon the expected future spot price minus dividends received during the holding period. The price of a commodity forward must also be based upon expectations, but there are several factors to consider. For example, based upon their physical qualities, some commodities are *storable* (e.g., metals) and the associated costs depend upon the physical characteristics of the commodity. Also, due to their physical nature, others are not storable (e.g., electricity, perishable foods).

Some commodities are also appropriate for *leasing*. That is, an investor without a current need purchases the commodity and then lends it out to others who do have a current need. Just as with the loan of any asset the lender requires a return, so a **lease rate** (i.e., required return) is established. For example, assume an investor uses cash and purchases a commodity. If a viable lease market exists for the commodity, the investor might lend it to someone. Since the investor used cash to acquire the commodity, he must charge a lease rate. Failing to do so would amount to an interest-free loan of the money tied up in the commodity.

Since commodity forward prices are based upon expected spot prices and expected spot prices are, in turn, dependent upon expected supply and demand forces, forward prices for commodities need not be constant from period to period. There are factors such as weather that can affect expected supply. For example, severe weather might be expected to reduce future coffee supplies, so the forward coffee price might incorporate the expected shortage into an increased forward price. Demand for a commodity can also be subject to change. For example, demand for electricity is not constant during the day nor is it constant across different seasons of the year or in different locations across the country. Estimating the expected spot price for a commodity, therefore, must utilize forecasts of all relevant factors.

For a given commodity on any trading day, several futures contracts will exist with varying maturity dates. The prices of the commodity futures contracts will differ with the different contract expiration dates. The set of futures prices for a given commodity is known as a **forward curve** or a **forward strip** on that particular day.

Assume that we do not know the forward price of the commodity and wish to derive it. A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time T.

The total cost at time 0 is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, where r represents the risk-free rate of return. The forward contract does not have any initial cash flows at time 0. The payoff at time T will be the payoff from the forward contract $(S_T - F_{0,T})$ plus the payoff from the bond $(F_{0,T})$:

$$S_T - F_{0,T} + F_{0,T} = S_T$$

where:
S_T = spot price of the commodity at time T

The present value of the expected spot price at time T is $E(S_T)e^{-\alpha T}$, where α represents the discount rate for the S_T cash flow at time T. This amount is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, because both represent the amount you would pay today to receive the commodity at time T. This equality is expressed in the following equation:

$$e^{-rT}F_{0,T} = E(S_T)e^{-\alpha T}$$

This equation illustrates that when using a risk-free discount rate, the discounted commodity forward price at time T is equivalent to the present value of a unit of commodity received at time T.

Multiplying each side of the equation by e^{rT} allows us to express the commodity forward price as follows:

$$F_{0,T} = E(S_T)e^{(r - \alpha)T}$$

Thus, the forward price today is a biased estimate of the expected commodity spot price at time T. The bias is a function of the risk premium on the commodity, $r - \alpha$. This equation is used to calculate the net present value (NPV) of commodities with available forward prices.

Commodity Arbitrage

LO 44.3: Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit.

A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

- Borrow money for the term of the contract at market interest rates.
- Buy the underlying commodity at the spot price.
- Sell a futures contract at the current futures price.

At contract expiration:

- Deliver the commodity and receive the futures contract price.
- Repay the loan plus interest.

If the futures contract is overpriced, this 5-step transaction will generate a riskless profit. The futures contract is overpriced if the actual market price is greater than the no-arbitrage price.

Example: Futures cash-and-carry arbitrage

Assume the spot price of gold is \$900/oz., that the 1-year futures price is \$975/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. **Calculate** the arbitrage profit.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at \$975. That means the futures contract is overpriced, so we should conduct cash and carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow \$900 to fund the purchase of gold, we must repay \$965.

Today		*1 year from today*	
Spot price of gold	\$900		
Futures price of gold	\$975		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Short futures	\$0	Settle short position by delivering gold	+\$975
Buy gold in spot market	–\$900		
Borrow at 7%	+\$900	Repay loan	–\$965
Total cash flow	\$0	Total cash flow = arbitrage profit	+\$10

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or \$975 – \$965 = \$10. Notice that this profit is equal to the difference between the actual futures price of \$975 and the no-arbitrage price of \$965.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

This is **reverse cash-and-carry arbitrage**. The steps in reverse cash-and-carry arbitrage are as follows.

At the initiation of the contract:

- Sell commodity short.
- Lend short sale proceeds at market interest rates.
- Buy futures contract at market price.

At contract expiration:

- Collect loan proceeds.
- Take delivery of the commodity for the futures price and cover the short sale commitment.

Example: Futures reverse cash-and-carry arbitrage

Assume gold is priced at \$900/oz., that the 1-year futures price is \$950/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. **Calculate** the profits from arbitrage.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at \$950. That means the futures contract is underpriced, so we should conduct reverse cash and carry arbitrage by going long in the futures contract, shorting gold, and investing the short-sale proceeds:

Today		*1 year from today*	
Spot price of gold	\$900		
Futures price of gold	\$950		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Long futures	\$0	Settle long position by buying gold	–\$950
Short gold	+\$900	Deliver gold to close short position	
Invest short-sale proceeds at 7%	–\$900	Receive investments proceeds	+\$965
Total cash flow	\$0	Total cash flow = arbitrage profit	+\$15

The riskless profit is equal to the loan proceeds less the futures contract payment, or \$965 – \$950 = \$15.

Professor's Note: It may help to remember "buy low, sell high." If the futures price is "too high," sell the future and buy the spot. If the futures price is "too low," buy the future and sell the spot.

Lease Rates

LO 44.4: Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price. The lease rate in the pricing of a commodity forward is very similar to the dividend payment in a financial forward.

A no-arbitrage price can be established if there is an active lending market for a commodity. A commodity lender can earn a return, the lease rate, by buying a commodity and immediately selling it forward. The amount a commodity borrower is willing to pay must equal the amount the lender requires in return for lending out the commodity for time T. This interest or lease amount is an important factor in establishing the forward price for the commodity.

The commodity forward price for time T with an active lease market is expressed as:

$$F_{0,T} = S_0 e^{(r - \delta_l)T}$$

where:
S_0 = commodity current spot price
$r - \delta_l$ = risk-free rate less the lease rate

The lease rate, δ_l, is income earned only if the commodity is loaned out.

Example: Pricing a commodity forward with a lease payment

Calculate the 12-month forward price for a bushel of corn that has a spot price of \$5 and an annual lease rate of 7%. The appropriate continuously compounding annual risk-free rate for the commodity is equivalent to 9%.

Answer:

We can determine the 12-month forward price as follows:

$$F_{0,T} = (S_0)e^{(r - \delta_l)T} = \$5 \times e^{(0.09 - 0.07)} = \$5.101$$

To further illustrate that this relationship must hold, consider the following no-arbitrage example.

Example: No-arbitrage for a commodity forward

Assume there is an active lending market for a bushel of corn. If no-arbitrage positions exist, **calculate** the forward price of a bushel of corn in one year if the lease rate is equal to 9%, the effective annual risk-free rate is equal to 9%, and the expected spot price in one year is equal to $2/bushel of corn.

Answer:

Figure 1 represents a no-arbitrage opportunity for a bushel of corn. An investor could borrow money at the risk-free rate of 9% to purchase a bushel of corn and short sell it forward. The investor immediately lends the bushel of corn out at a lease rate of 9%. At the end of the lease period, T_1, the individual would pay back the loan with interest at $2.18, sell the corn at $2.00, and receive the lease payment of $0.18. In order for a no-arbitrage position to exist, the forward price, $F_{0,1}$, must be equal to the expected spot price of $2.00.

Figure 1: No-Arbitrage Opportunity on Bushel of Corn

Transaction	*Time = T_0*	*Time = T_1*
Borrow @ 9%	$2.00	$(2.18)
Buy a bushel of corn	$(2.00)	$2.00
Lend bushel of corn	$0	$0.18
Short forward @ $2	$0	$F_{0,1}$–$2
Total	$0	$F_{0,1}$–$2

Contango and Backwardation

An upward-sloping forward curve indicates that forward prices more distant in time are higher than current forward prices. The market is described as being in **contango** with an upward-sloping forward curve. A contango commodity market occurs when the lease rate is less than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta_l)T}$, if $r > \delta_l$, the forward price must be greater than the spot price.

The market is described as being in **backwardation** with a downward-sloping forward curve. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0 e^{(r - \delta_l)T}$, if $r < \delta_l$, the forward price must be less than the spot price.

Storage Costs

LO 44.5: Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.

LO 44.6: Compute the forward price of a commodity with storage costs.

When holding a commodity requires storage costs, *the forward price must be greater than the spot price* to compensate for the physical storage costs (i.e., costs associated with constructing and maintaining a storage facility) and financial storage costs (i.e., interest). The owner of a commodity can either sell it today for a price of S_0 or for delivery at time T at the forward price. If the owner sells it at a forward price, this is known as *cash-and-carry* (as we saw in LO 44.3) because the seller receives the cash but must store (i.e., carry) the commodity until the delivery date. The market in which a commodity is stored is referred to as a **carry market**. The owner will only store the commodity if the forward price is greater than or equal to the expected spot price plus storage costs. This is represented mathematically as:

$$F_{0T} \geq S_0 e^{rT} + \lambda(0,T)$$

where:
$\lambda(0,T)$ = FV of storage costs for one unit of the commodity from time 0 to T

If *storage costs are paid continuously and are proportional to the value of the commodity*, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r + \lambda)T}$$

where:
λ = continuous annual storage cost proportional to the value of the commodity

Example: Commodity forward pricing with storage costs and effective interest

Calculate the 3-month forward price for a bushel of soybeans if the current spot price is \$3/bushel, the effective monthly interest rate is 1%, and the monthly storage costs are \$0.04/bushel.

Answer:

First, calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$$\$0.04 + \$0.04(1.01) + \$0.04(1.01)^2 = \$0.1212$$

The amount of \$0.1212 represents the three months storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0 e^{rT} + \lambda(0,T) \approx \$3.00(1.01^3) + \$0.1212 = \$3.0909 + \$0.1212 = \$3.2121$$

Professor's Note: Notice the approximation used in the previous example: $F_{0,T} = S_0 e^{rT} \approx S_0 \times (1 + r)^T$*. Using either approach will produce similar results.*

Convenience Yield

If the owners of the commodity need the commodity for their business, holding physical inventory of the commodity creates value. For example, assume a manufacturer requires a specific commodity as a raw material. To reduce the risk of running out of inventory and slowing down production, excess inventory is held by the manufacturer. This reduces the risk of idle machines and workers. In the event that the excess inventory is not needed, it can always be sold. Holding an excess amount of a commodity for a non-monetary return is referred to as **convenience yield.**

A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity. The forward price including a convenience yield is calculated as follows:

$$F_{0,T} \geq S_0 e^{(r + \lambda - c)T}$$

where:
c = continuously compounded convenience yield, proportional to the value of the commodity

For the investor who does not earn the convenience yield, cash-and-carry arbitrage implies that:

$$F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

Example: Impact of convenience yield on the no-arbitrage cash-and-carry commodity forward pricing range

Suppose the owner of a commodity decides to lend out the commodity. The commodity has a continuously compounded convenience yield of *c*, proportional to the value of the commodity. **Determine** which range of prices must represent the no-arbitrage cash-and-carry opportunity for an investor who recognizes a convenience yield.

Answer:

The owner of a commodity is able to create a range of no-arbitrage prices as follows:

$$S_0 e^{(r + \lambda - c)T} \leq F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

The upper bound depends on storage costs but not on the convenience yield. The lower bound adjusts for the convenience yield and therefore explains why forward prices may appear lower at times when the convenience yield is accounted for.

Comparing Lease Rates, Storage Costs, and Convenience Yield

LO 44.7: Compare the lease rate with the convenience yield.

Here is a handy guide for relating forward and spot commodity prices on the exam. Start with the basic expression relating forward and spot prices:

$$F_{0,T} = S_0 e^{rT}$$

This expression says that if there are no costs or benefits associated with buying and holding the commodity, the forward price is just the spot price compounded at the risk-free rate over the holding period.

If there are benefits (e.g., lease rates, convenience yield) to buying the commodity today, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield:

$$F_{0,T} = S_0 e^{(r - c)T} < S_0 e^{rT}$$

where c = the convenience yield, or

$$F_{0,T} = S_0 e^{(r - \delta)T} < S_0 e^{rT}$$

where δ = the lease rate

If there are costs, such as storage costs, associated with purchasing the commodity today, the forward price is increased by the cost:

$$F_{0,T} = S_0 e^{(r + \lambda)T} > S_0 e^{rT}$$

where λ = the storage costs

Of course, there can be combinations of costs and benefits, so be sure to increase the exponent for costs and reduce it for benefits:

$$F_{0,T} = S_0 e^{(r + \lambda - c)T}$$

In the equation above, the lease rate is equal to storage costs minus the convenience yield.

Commodity Characteristics

LO 44.8: Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices.

Certain commodities exhibit unique properties that impact their forward price. For example, gold, corn, electricity, natural gas, and oil are all commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and seasonal

demand. These differences are reflected in lease rates, storage costs, and convenience yields that influence the commodity forward prices and the shape of the forward curves.

Gold Forward Price Factors

Because gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out. When a positive lease rate is present, the synthetic gold is preferred to physically holding the gold because the lease rate represents the cost of holding the gold without lending it.

The value of gold is also influenced by the cost of production. The present value of gold received in the future is simply the present value of the forward price computed at the risk-free rate of return. The present value of gold production is calculated as follows:

$$\text{PV of gold production} = \sum_{i=1}^{n} n_{t_i}\left[F_{0,t_i} - x(t_i)\right] e^{-r(0,t_i)t_i}$$

where:
n_{t_i} = amount of ounces of gold we expect to extract, with an extraction cost of $x(t_i)$

Under this framework, the gold mine is assumed to operate the entire time, and production is known with certainty.

Corn Forward Price Factors

Corn is an example of a commodity with seasonal production and a constant demand. Corn is produced every fall, but it is consumed throughout the year. In order to meet consumption needs, corn must be stored. Thus, interest and storage costs need to be considered. The price of the corn will fall as it is being harvested and then rise to reflect the cost of storage over the next 12 months until it is harvested again. Thus, the forward curve is increasing until harvest time, and it drops sharply and slopes upward again after harvest time is over.

Example: Corn commodity pricing with storage costs

Suppose the spot price today for a bushel of corn is \$2.25, the continuously compounded interest rate is 5.5%, and the storage cost is 2.0% per month. **Calculate** the 6-month forward price.

Answer:

$$F_{0,0.5} = \$2.25 \times e^{(0.00458 + 0.02)6} = \$2.25 \times 1.15893 = \$2.61$$

Professor's Note: The 0.458% used for the monthly interest rate is the annual rate divided by 12.

Electricity Price Factors

As previously mentioned, electricity is not a storable commodity. Once it is produced, it must be used or it will likely go to waste. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season. Given the non-storability characteristic of electricity, its price is set by demand and supply at a given point in time. Since arbitrage opportunities do not exist with electricity (i.e., the inability to buy electricity during one season and sell it during another season) futures prices on electricity will vary much more during the trading day than financial futures.

Natural Gas Forward Price Factors

Natural gas is an example of a commodity with constant production but seasonal demand. Natural gas is expensive to store, and demand in the United States peaks during high periods of use in the winter months. In addition, the price of natural gas is different for various regions due to high international transportation costs. Storage is at its peak in the fall just prior to the peak demand. Therefore, the forward curve rises steadily in the fall.

Example: Calculation of natural gas forward price with storage costs

Calculate the natural gas implied storage cost for the month of October if the October 2005 spot price is 4.071, the annual risk-free rate of interest is 6%, and the November forward price is 4.157.

Answer:

$$\$4.157 = \$4.071e^{0.005} + \lambda_{\text{Oct2005}}$$

$$\$4.157 = \$4.091 + \lambda_{\text{Oct2005}}$$

$$\$4.157 - \$4.091 = \lambda_{\text{Oct2005}}$$

$$\$0.066 = \lambda_{\text{Oct2005}}$$

Oil Forward Price Factors

The physical characteristics of oil make it is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. In addition, demand is high in one hemisphere when it is low in the other. Lower transportation costs and more constant worldwide demand causes the long-run forward price to be more stable. In the short-run, supply and demand shocks cause more volatile prices because supply is fixed. For example, the Organization of Petroleum Exporting Countries (OPEC) may decrease supply to increase prices by causing a shortage in the short run. Supply and demand adjust to price changes in the long run.

Commodity Spread

LO 44.9: Compute a commodity spread.

A **commodity spread** results from a commodity that is an input in the production process of other commodities. For example, soybeans are used in the production of soybean meal and soybean oil. A trader creates a **crush spread** by holding a long (short) position in soybeans and a short (long) position in soybean meal and soybean oil.

Similarly, oil can be refined to produce different types of petroleum products such as heating oil, kerosene, or gasoline. This process is known as "cracking," and thus the difference in prices of crude oil, heating oil, and gasoline is known as a **crack spread**. For example, seven gallons of crude oil may be used to produce four gallons of gasoline and three gallons of heating oil. Commodity traders refer to the crack spread as 7-4-3, reflecting the seven gallons of crude oil, four gallons of gasoline, and three gallons of heating oil. Thus, an oil refiner could lock in the price of the crude oil input and the finished good outputs by an appropriate crack spread reflecting the refining process. However, this is not a perfect hedge because there are other outputs that can be produced such as jet fuel and kerosene.

Example: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently \$30/barrel. The 2-month future prices for gasoline and heating oil are \$41/barrel and \$31.50/barrel, respectively. **Calculate** the 5-3-2 crack (commodity) spread.

Answer:

The 5-3-2 spread tells us the amount of profit that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

profit for a 5-3-2 spread =
$(3 \times \$41) + (2 \times \$31.50) - (5 \times \$30) = \$123 + \$63 - \$150 = \$36$ for five barrels, or
$\$36 / 5$ barrels = \$7.20/barrel

Professor's Note: There is no calculation for interest adjustment in this example.

Basis Risk

LO 44.10: Explain how basis risk can occur when hedging commodity price exposure.

As you may recall, **basis** is the difference between the spot price (or rate) and the price (or rate) of the futures contract used to hedge. If the values of both move together perfectly, an

investor long or short the asset can lock in a return or value by selling or buying futures, respectively.

Professor's Note: When you expect to receive the commodity in the future, we say you are long the commodity and you will hedge the value of the expected commodity by selling the corresponding futures contracts. If you will deliver the commodity in the future without first owning the commodity, you are short, and you will hedge by taking a long position in the corresponding futures contracts.

Any time the values of the spot and futures contracts do not move together perfectly, the hedger faces **basis risk**. An example with financial futures is using a basket currency futures contract to hedge the value of a transaction in an emerging market. Since the hedged asset (i.e., the emerging market currency) and the underlying in the futures contract are not identical, there is risk associated with changes in their relative values. Also, if the financial futures contract must be rolled over, or if it matures after the delivery date, this adds to the basis risk.

Since there are storage and transportation costs associated with commodities, hedgers face more concerns. As with financial futures, every commodity futures contract specifies a delivery amount and a delivery date. In addition, however, every commodity futures contract specifies a delivery *location* and the deliverable *grade* (i.e., quality). For example, an investor planning to receive oil in New York City might use NYMEX futures, which specify delivery in Oklahoma. At the producer level, an Iowa corn farmer might use CBOT corn futures, which specify delivery in Chicago.

Strip Hedge vs. Stack Hedge

LO 44.11: Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management.

An oil producer may enter into a contract to supply a fixed amount of barrels of oil per month at a fixed price. The oil producer could set up a **strip hedge** by buying futures contracts that match the maturity and quantity for every month of the obligation.

To help reduce transaction costs, the oil producer might instead utilize a **stack hedge**. To form a stack hedge, the oil producer would enter into a one-month futures contract equaling the total value of the year's promised deliveries. As transactions costs are less for short-term (e.g., one-month) contracts, the total cost of implementing this strategy is less than for a comparable strip hedge. At the end of the first month, the producer rolls into the next one-month contract, and so forth, each month setting the total amount of the contract equal to the remaining promised deliveries. This strategy of continually rolling into the next near-term contract is referred to as **stack and roll**.

A stack hedge has the advantage when near-term contracts are more readily available due to heavier volume and more liquidity. Another advantage of near-term contracts is that distant futures on commodities often have wider bid-ask spreads and therefore larger transaction costs. In addition, an oil producer may prefer a stack hedge in order to speculate on the shape of the forward curve. For example, assume the forward curve looks unusually steep.

The oil producer would then enter into a stacked hedge with a large near-term contract. If the forward curve later flattens, the oil producer locks in all the oil at a relatively cheap near-term price compared to the more expensive futures using the strip strategy.

Example: Creation of a strip or stack hedge

Determine how an oil producer could hedge the risk of an agreement to supply 150,000 barrels of oil each month for a year at a fixed price.

Answer:

The oil producer could enter into a strip hedge by obtaining a long futures contract position for every month of the year for 150,000 barrels.

Alternatively, the oil producer could create a long position of a near-term futures contract for a little less than 1,800,000 barrels. At the end of the month, the oil producer would enter into a new near-term futures contract for a smaller amount representing the present value of future deliveries.

Cross Hedging

LO 44.12: Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives.

In some cases, a futures contract with an underlying instrument that is exactly the same as the position to be hedged will not exist. For example, there are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel requires a **cross hedge**. Some firms hedge the cost of jet fuel with crude oil futures while others hedge using a combination of crude oil and heating oil futures. Three factors are relevant when making a cross hedge decision:

- The liquidity of the futures contract (since delivery may not be an option).
- The correlation between the underlying for the futures contract and the asset(s) being hedged.
- The maturity of the futures contract.

Each of these factors has an impact on the effectiveness of the hedge. The liquidity of the cross hedge is important in order for the portfolio manager to quickly unwind the futures obligation. Thus, the manager should try to choose among liquid instruments to find the futures contract whose maturity most closely matches that of the horizon of the hedged position.

To illustrate the concept of cross hedging, consider a firm that uses crude oil futures to hedge jet fuel prices. The payoff from this type of hedge will depend on both the change in jet fuel prices and the change in oil futures prices. Thus, the number of crude oil futures contracts required is estimated using regression analysis, where the change in jet fuel prices is dependent on the change in oil futures prices. The slope coefficient from the regression

results will provide the portfolio manager with hedge ratio information regarding the degree that crude oil price changes affect the price of jet fuel.

A cross hedge is also applied when firms use **weather derivatives.** Weather risk is a business risk that is faced by agricultural firms as well as many firms involved with providing recreational services. It refers to any financial losses, explicit and implicit, that a firm faces from changes in the weather.

Utility companies use weather derivatives, which are based on "degree days," to hedge the cost of energy purchases. Much of the energy supplied by utilities is used for heating or cooling with variations in demand directly correlated with weather patterns. Demand can rise and fall dramatically in conjunction with the weather experienced in the areas that the utilities service.

Utilities can use derivatives with payoffs based on the weather experienced at weather stations that are representative of the areas that they serve. For example, a utility located in the northeast U.S. contracts for energy needs based on average weather experienced over previous years and predictions for the coming year. Unhedged, the utility would leave itself exposed to rising prices from energy producers in the event that the coming winter is far worse than predicted.

If hedging with weather derivatives (specifically weather options), and the winter were worse than expected (have more heating degree days than the strike value of the contract), the utility would receive the specified payment. If the winter were milder than expected, the contract would expire worthless. The actual measurements are from specified U.S. government sites in the areas specified by the contract.

The use of weather derivatives by other investors is growing, but one of the biggest problems is basis risk. That is, it is difficult to accurately match up the exposure of other assets to the weather with that specified by the contracts. Other than large-scale exposure, such as that experienced by utilities, many producers are much more susceptible to more local variations. For instance, a large farming operation has exposure to the rain falling on its own fields and may suffer losses from too much or too little rain. The rain on its fields may not have a high correlation with the rain experienced at the weather station 50 miles away.

Key Concepts

LO 44.1

When holding a commodity requires storage costs, the forward price must be greater than the spot price to compensate for the physical storage costs and financial storage costs.

The market in which a commodity is stored is referred to as a carry market.

A lease rate is the amount of interest a lender of a commodity requires.

Holding an excess amount of a commodity for a non-monetary return is referred to as convenience yield.

LO 44.2

The commodity forward price today is defined as a biased estimate of the expected spot commodity price at time T as follows:

$$F_{0,T} = E(S_T)e^{(\text{risk-free rate} - \text{discount rate})T}$$

LO 44.3

The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

Step 1: Borrow money for the term of the contract at market interest rates.
Step 2: Buy the underlying commodity at the spot price.
Step 3: Sell a futures contract at the current futures price.

At contract expiration:

Step 1: Deliver the commodity and receive the futures contract price.
Step 2: Repay the loan plus interest.

LO 44.4

The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. If an active lease market exists for a commodity, a commodity lender can earn the lease rate by buying a commodity and immediately selling it forward.

The commodity market is in contango with an upward-sloping forward curve when the lease rate is less than the risk-free rate. The market is in backwardation with a downward-sloping forward curve when the lease rate is greater than the risk-free rate.

LO 44.5

Holding an excess physical inventory of the commodity creates non-monetary value for commodity owners who require the commodity as a production input. This is referred to as convenience yield, and the forward price including a convenience yield is calculated as:

$F_{0,T} \geq S_0 e^{(r+\lambda-c)T}$, where c is the continuously compounded convenience yield, proportional to the value of the commodity

The owner of a commodity who uses the commodity in production is able to create a range of no-arbitrage prices as follows:

$$S_0 e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0 e^{(r+\lambda)T}$$

LO 44.6

A commodity owner will only store the commodity if the forward price is greater than or equal to the spot price plus the future storage costs as follows:

$F_{0,T} \geq S_0 e^{rT} + \lambda(0,T)$, where $\lambda(0,T)$ represents the future value of storage costs for one unit of the commodity from time 0 to T.

If storage costs are paid continuously and are proportional to the value of the commodity, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r+\lambda)T}$$

LO 44.7

If there are benefits to buying the commodity, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield.

LO 44.8

Since gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out.

Corn is an example of a commodity with seasonal production and a constant demand.

Electricity is not a storable commodity. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season.

Natural gas is an example of a commodity with constant production but seasonal demand.

Oil is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. Supply and demand adjust to price changes in the long run.

LO 44.9

A commodity spread results from a commodity that is an input in the production process of other commodities. For example, a 7-4-3 crack spread refers to the profit for holding four gasoline futures plus three heating oil futures less seven crude oil futures.

LO 44.10

Basis risk results from the inability to create a perfect hedge due to differences in the commodities with respect to timing, grade, storage costs, and/or transportation costs.

LO 44.11

A strip hedge is created by buying futures contracts that match the maturity and quantity for every month of the obligation. A stack hedge is created by buying a futures contract with a single maturity based on the present value of the future obligations. Advantages of the stack hedge are the availability and liquidity of near-term contracts and narrower bid-ask spreads for near-term contracts.

LO 44.12

There are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel costs requires a cross hedge (e.g., hedge with crude oil futures). A cross hedge is also applied when firms use weather derivatives.

LO 44.13

A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time T.

Concept Checkers

1. Which of the following statements regarding lease rates is(are) true? The lease rate is:

   I. the amount of return the investor requires to buy and then lend a commodity.
   II. very similar to the dividend payment in a financial forward.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Suppose there is an active lending market for a bushel of soybeans (which has a current spot price of $4/bushel). If the annual lease rate is equal to 7%, the effective annual risk-free rate is equal to 7%, and the expected spot price in one year is equal to $4/bushel of soybeans, how could an investor create an arbitrage opportunity? An individual could:
   A. borrow money at 7% and purchase a bushel of soybeans and sell it forward.
   B. borrow a bushel of soybeans and sell a bushel of soybeans at the spot price and buy a long forward.
   C. sell a bushel of soybeans at the forward price and lend the money at the risk-free rate.
   D. go long in soybean forward contracts, short in soybean spot prices, and lend the excess proceeds at the risk-free rate.

3. What is the 3-month forward price for a bushel of corn if the current spot price for corn is $3/bushel, the effective monthly interest rate is 1.5%, and the monthly storage costs are $0.03/bushel?
   A. $3.18.
   B. $3.23.
   C. $3.29.
   D. $3.31.

4. Suppose we plan on buying crude oil in one month to produce gasoline and heating oil for sale in two months. The 1-month future price for crude oil is currently $42.5/barrel. The 2-month future prices for gasoline and heating oil are $45/barrel and $43.50/barrel, respectively. What is the 7-5-2 crack (commodity) spread?
   A. $2.07/barrel.
   B. $6.00/barrel.
   C. $14.50/barrel.
   D. $22.09/barrel.

5. Which of the following statements is an example of basis risk? Purchasing:
   A. an oil contract with delivery in a different geographical region.
   B. a commodity with a desired distant delivery with near-term contracts.
   C. a eurodollar contract, due to lack of commodity futures.
   D. All of the above statements are correct.

Concept Checker Answers

1. C A *lease rate* is the amount of interest a lender of a commodity requires. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate in the pricing of a commodity future is very similar to the dividend payment in a financial forward.

2. A An individual could borrow money at the risk-free rate of 7% to purchase a bushel of soybeans and sell it forward. The individual immediately lends the bushel of soybeans out at a lease rate of 7%. At the end of the lease period, T_1, the individual would pay back the loan with interest at $4.28, sell the soybeans at $4.00, and receive the lease payment of $0.28. In order for a no-arbitrage position to exist, the forward price, $F_{0,1}$, must be equal to the expected spot price of $4.00. An arbitrage position exists if the forward price is not equivalent to the expected spot price.

No-Arbitrage Opportunity on Bushel of Soybeans

Transaction	*Time* = T_0	*Time* = T_1
Borrow @ 7%	$4.00	($4.28)
Buy a bushel of soybeans	($4.00)	$4.00
Lend bushel of soybeans	$0	$0.28
Short forward @ $4	$0	$F_{0,1}$–$4
Total	$0	$F_{0,1}$–$4

3. B First calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$$\$0.03 + \$0.03(1.015) + \$0.03(1.015)^2 = \$0.0914$$

The amount of $0.0914 represents the 3-month storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0e^{rT} + \lambda(0,T) \approx \$3.00(1.015^3) + \$0.0914 = \$3.1370 + \$0.0914 = \$3.23$$

4. A The 7-5-2 spread tells us the amount of profit that can be locked in by buying seven barrels of oil and producing five barrels of gasoline and two barrels of heating oil.

Profit for a 7-5-2 spread =

(5 × $45) + (2 × $43.50) – (7 × $42.5) = $225 + $87 – $297.5 = $14.50 for seven barrels, or $14.5 / 7 barrels = $2.07/barrel.

5. D All are examples of basis risk, which results from the inability of commodities to create a perfect hedge. Differences due to timing, grade, storage costs, or transportation costs create basis risk.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Foreign Exchange Risk

Topic 45

Exam Focus

Exposure to foreign exchange risks is a natural result of the globalization of financial institutions. These risks arise when foreign currency trading and/or foreign asset-liability positions are mismatched in individual currencies. Unexpected volatility can generate significant losses for the firm, which could, in turn, threaten profitability or even solvency. These risks can be mitigated by direct hedging through matching foreign asset-liability books of business, hedging through forward contracts, and through foreign asset and liability portfolio diversification.

Sources of Foreign Exchange Risk

LO 45.1: Calculate a financial institution's overall foreign exchange exposure.

LO 45.2: Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk.

LO 45.3: Calculate a financial institution's potential dollar gain or loss exposure to a particular currency.

Large financial institutions (banks) frequently take significant positions in foreign currency assets and liabilities as a result of their foreign exchange trading activities. When looking at such financial institutions' currency trading activities, the aggregate position size in a particular currency may look extremely large; however, since buys and sells will offset one another in terms of exposure, the net exposure to the currency may actually be quite small.

A bank's actual exposure to any given currency can be measured by the **net position exposure.** Net exposure is the extent to which a bank is net long (or *positive*) or net short (or *negative*) in a given currency. For example, a bank's net euro (EUR) exposure would be:

$$\text{net EUR exposure} = (\text{EUR assets} - \text{EUR liabilities}) + (\text{EUR bought} - \text{EUR sold})$$

$$\text{net EUR exposure} = \text{net EUR assets} + \text{net EUR bought}$$

A **positive net exposure** position means that we are *net long in a currency.* In other words, we hold more assets than liabilities in a given currency. In this instance, the financial institution faces the risk that the foreign currency will *fall* in value against the domestic currency.

A **negative net exposure** position means that we are *net short in a currency.* The financial institution faces the risk that the foreign currency will *rise* in value against the domestic currency.

Therefore, if a U.S. financial institution fails to maintain a balanced position in a currency where assets (purchases) are exactly offset by liabilities (sales), the institution will be exposed to variations in the foreign exchange (FX) rate of that currency against the U.S. dollar. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

Foreign Trading Activities

LO 45.4: Identify and describe the different types of foreign exchange trading activities.

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

1. Enabling customers to participate in international commercial business transactions.
2. Enabling customers to take positions in real or financial foreign investments. Note that a financial institution may also transact in foreign currencies to take positions in real or financial foreign investments for its own portfolio.
3. Offsetting exposure in a given currency for hedging purposes.
4. Speculating on foreign currencies in search of profit by forecasting and/or anticipating futures FX rate movements.

When a bank is buying or selling a foreign currency for the purpose of either allowing its customers to participate in international commercial business transactions or investing in real or financial foreign investments, the bank typically serves as an agent for the customers (receives a fee) and does not assume the FX risk itself.

When a bank is buying or selling a currency for hedging purposes, this will reduce FX exposure.

The fourth activity, trading foreign currencies with the intent to profit by anticipating future foreign currency rate movements, relates to open positions that are taken for speculative purposes and represents an unhedged position in a given currency. These speculative trades are usually made directly with other financial institutions or arranged through FX specialist brokers.

Currency spot trades are the most frequently executed speculative trades. The financial institution seeks to earn a profit on the difference between the buy and sell prices or on movements in the bid-ask spreads over time. Speculative positions can also be taken in FX forward contracts, futures, and options.

Sources of Profits and Losses on Foreign Exchange Trading

LO 45.5: Identify the sources of foreign exchange trading gains and losses.

LO 45.6: Calculate the potential gain or loss from a foreign currency denominated investment.

Most returns on FX trading arise from speculation in currencies or taking an unhedged position in a particular currency. Financial institutions also earn fees as a secondary source of revenues. These revenues are earned from market-making activities and/or from acting as agents for retail or wholesale customers.

Mismatched Foreign Asset and Liability Positions

A financial institution can also have foreign exchange exposure due to mismatches between foreign financial asset and liability portfolios. The following example shows the exposure resulting from such a mismatch.

Example: Foreign investment returns

Figure 1: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yielding 8% USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD100 million U.S. CDs, 1-year maturity, in USD, yielding 6%

This firm has matched the duration of its assets and liabilities ($D_A = D_L = 1$ year) but has mismatched the currency composition of its portfolio. Note that the firm would earn a positive spread of 2% (8% – 6%) from investing domestically. In order to invest in Switzerland, this firm decides to take 50% of its $100 million and make 1-year Swiss loans while keeping 50% to make U.S. dollar loans. What transactions must the firm undertake to make the CHF-denominated loan (assuming the FX position is not hedged)?

Answer:

1. Sell USD50 million for CHF on the spot currency markets at the beginning of the year. If the exchange rate is USD1.70 to 1 CHF, this yields USD50,000,000 / 1.7 = CHF29,411,765.
2. Use the CHF29,411,765 to make 1-year Swiss loans at a 13% interest rate.
3. At the end of the one year, CHF revenue from these loans will be CHF29,411,765(1.13) = CHF33,235,294 (assuming no default).
4. At the end of the year, repatriate these funds back to the United States. In other words, the U.S. bank will sell CHF33,235,294 in the FX market at the spot exchange rate that exists at the end of the year.

In this example, we assume the spot FX rate has not changed over the 1-year period and remains at USD1.70/CHF. The dollar proceeds from the Swiss investment would be:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.70 / \text{CHF} = \text{USD}56{,}500{,}000, \text{ for a return of:}$$

$$\frac{\text{USD}56{,}500{,}000 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 13.0\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.13) = 0.105 \text{ or } 10.5\%$$

This exceeds the cost of the bank CDs by 4.5% (=10.5% – 6.0%).

Example, continued:

Now, suppose that at the end of the year, the Swiss franc has *fallen* in value relative to the U.S. dollar. If the exchange rate is now USD1.55/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

The Swiss loan revenues at the end of one year equal:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.55 / \text{CHF} = \text{USD}51{,}514{,}706, \text{ for a return of:}$$

$$\frac{\text{USD}51{,}514{,}706 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 3.03\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.0303) = 0.0552 \text{ or } 5.52\%$$

Under this scenario, the bank would actually have a negative interest margin on its balance sheet investments of –0.48% since its cost of funds (COFs) is 6.0%.

Example, continued:

If the Swiss franc had *appreciated* against the dollar over the year, the bank would have generated a double benefit: (1) from the appreciation of the franc, and (2) from the higher yield on the domestic Swiss loans. If the exchange rate is now USD1.82/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.82 / \text{USD} = \text{USD}60{,}488{,}235, \text{ for a return of:}$$

$$\frac{\text{USD}60{,}488{,}235 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 20.98\%$$

The previous example illustrates an important concept. As with any investment, returns for the bank's portfolio are derived from differences between income and costs. However, foreign investing provides the additional dynamic of having profits or losses affected by changes in foreign exchange rates. There are two principle methods available to control the scale of FX exposure: on-balance-sheet hedging and off-balance-sheet hedging.

Balance Sheet Hedging

LO 45.7: Explain balance-sheet hedging with forwards.

On-Balance-Sheet Hedging

On-balance-sheet hedging is achieved when a financial institution has a matched maturity and currency foreign asset-liability book. Figure 2 is an illustration.

Figure 2: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yield 8%	USD50 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD50 million Swiss CDs, 1-year maturity, raised in CHF, yielding 10%

Using the data in Figure 2, we can examine the effects of the franc depreciating by the same amount as in the previous example:

1. The bank borrows USD50 million equivalent in Swiss francs for one year at an interest rate of 10%. At the exchange rate of USD1.70/CHF, this equates to USD50,000,000 / 1.70 = CHF29,411,765.
2. At the end of one year, the bank must pay back the Swiss franc CD holders their principal and interest: CHF29,411,765 × (1.10) = CHF32,352,941.
3. If the franc *depreciated* to USD1.55/CHF in the period, repayment in dollar terms would be CHF32,352,941 × USD1.55/CHF = USD 50,147,059, or a dollar cost of funds of 0.3%.
4. The bank makes CHF29,411,765 in loans at 13% for one year.
5. At the end of one year, the loans are repaid with interest. CHF29,411,765 (1.13) = CHF33,235,294, but at USD1.55/CHF, this equals only USD51,514,706 for a return of 3.03%.

At the end of the year, we would have the following.

Average return on assets:

$$(0.5)(0.08)+(0.5)(0.0303)=0.0552 \text{ or } 5.52\%$$
$$\text{U.S. asset return} + \text{CHF asset return} = \text{overall return}$$

Average cost of funds:

$$(0.5)(0.06)+(0.5)(0.003)=0.0315 \text{ or } 3.15\%$$
$$\text{U.S. cost of funds} + \text{CHF cost of funds} = \text{overall cost}$$

Net return:

$$5.52\% - 3.15\% = 2.37\%$$
$$\text{average return on assets} - \text{average cost of funds}$$

By directly matching foreign assets and liabilities, we can lock in a positive return or profit spread if exchange rates move in either direction over the investment period.

Off-Balance-Sheet Hedging

Rather than matching foreign assets with foreign liabilities, we may choose to remain unhedged on the balance sheet. If we do, we could hedge off-balance-sheet by taking a position in the forward market. This hedge would appear as a contingent off-balance-sheet claim as an item below the net income line.

Referring to the previous example, the function of the forward FX contract is to offset the uncertainty of the future spot rate on the CHF at the end of the investment horizon. A forward foreign exchange agreement involves the exchange of a foreign currency at some point in the future at an exchange rate that is determined today. Rather than repatriating CHF and exchanging them for USD at the end of the period at an unknown rate, the bank can enter into a contract to sell forward the *expected* principal and interest on the loan at the current known **forward exchange rate** for USD/CHF, with the delivery of Swiss francs to the buyer of the forward contract taking place at the end of the investment horizon. This method effectively removes the future spot exchange rate uncertainty that is related to investment returns on the Swiss loan. By using the data in Figure 2, we can illustrate how this technique would work.

Example: Hedging with forward contracts

Outline the transactions necessary for the financial institution to use an off-balance-sheet hedge for the asset-liability position described in Figure 2.

Answer:

The following transactions create the off-balance-sheet hedge.

1. The U.S. bank sells USD50 million for Swiss francs at the *spot* exchange rate *today* and receives USD50,000,000 / USD1.7/CHF = CHF29,411,765.

2. Immediately after the sale, the bank lends the CHF29,411,765 to a Swiss customer at 13% for one year.

3. In addition, the bank sells the expected principal and interest proceeds from the franc loan forward for U.S. dollars at today's forward rate (say, USD1.65/CHF) for 1-year delivery: (USD1.65 – USD1.70) / USD1.70 = –2.94%.

   The forward buyer of the francs will pay USD54,838,235 to the seller when the bank delivers the CHF33,235,294 proceeds of the loan to the financial institution seller.

$$\begin{aligned}\text{CHF29,411,765}(1.13) \times \text{USD1.65/CHF} &= \text{CHF33,235,294} \times \text{USD1.65/CHF}\\ &= \text{USD54,838,235}\end{aligned}$$

4. At the end of one year, the Swiss borrower repays the loan to the bank plus interest in Swiss francs (CHF33,235,294).

5. The bank gives the CHF33,235,294 to the buyer of the 1-year forward contract and receives USD54,838,235.

By using this method, the bank knows it has locked in a guaranteed return of 9.68% on the Swiss franc (assuming, of course, the loan will not default and the forward buyer does not renege on the forward contract).

$$\frac{\text{USD54,838,235} - \text{USD50,000,000}}{\text{USD50,000,000}} = 0.0968 = 9.68\%$$

The overall expected return on the bank's asset portfolio would then be:

$$(0.5)(0.08) + (0.5)(0.0968) = 8.84\%$$

Regardless of spot exchange rate fluctuations over the year, the bank has locked in a risk-free return spread of 2.84% (8.84% return – 6% cost of funds) over the cost of funds for the bank's CDs.

LO 45.8: Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates.

Because the hedged Swiss loans offer a higher return than the U.S. loans, it makes sense for the bank to focus its activities on making hedged Swiss loans. However, as more is invested in Swiss loans, the bank must buy more Swiss francs. This will continually reduce the forward rate spread until no additional profits could be made by making the forward contract-hedged investments.

As the bank moves into more Swiss loans, the spot exchange rate for buying francs will rise. In equilibrium, the forward exchange rate would have to fall to completely eliminate the attractiveness of the Swiss investments.

This relationship is called **interest rate parity** (IRP) since the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. In other words, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

For the exam, you should know that the exact IRP equation using direct quotes is:

$$\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^{T}$$

where:
r_{DC} = domestic currency rate
r_{FC} = foreign currency rate

If this equality does not hold, an arbitrage opportunity exists. To remember this formula, note that when the forward and spot rates are expressed as direct quotes (DC/FC), right-hand side of the equation also has the domestic (interest rate) in the numerator and the foreign (interest rate) in the denominator.

If we expressed the forward and spot rates as indirect quotes (FC/DC), then the right-hand side of the equation would have the foreign (interest rate) in the numerator and the domestic (interest rate) in the denominator. So it's either domestic over foreign for everything, or foreign over domestic for everything.

IRP can also be stated using continuously compounded rates as follows:

$$\text{forward} = \text{spot} \times e^{(r_{DC} - r_{FC})T}$$

Example: Interest rate parity

Suppose you can invest in NZD at 5.127%, or you can invest in Swiss francs at 5.5%. You are a resident of New Zealand, and the current spot rate is 0.79005 NZD/CHF. **Calculate** the 1-year forward rate expressed in NZD/CHF.

Answer:

$$\text{forward(DC / FC)} = \text{spot(DC / FC)}\left|\frac{(1+r_{DC})}{(1+r_{FC})}\right| = 0.79005\left(\frac{1.05127}{1.055}\right) = 0.78726$$

Professor's Note: Notice here that the NZD/CHF rate fell from 0.79005 to 0.78726. This implies that it now takes fewer NZD to buy one CHF. So, in other words, the New Zealand dollar has appreciated relative to the Swiss franc. Consequently, the Swiss franc has depreciated relative to the New Zealand dollar.

Diversification in Multicurrency Foreign Asset-Liability Positions

LO 45.9: Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk.

LO 45.10: Describe the relationship between nominal and real interest rates.

Our previous examples have used matched and mismatched asset-liability portfolios that involve only one foreign currency. In reality, most financial institutions hold positions in many different currencies in their asset-liability portfolios. Since currencies may be less than perfectly correlated, diversification across several asset and liability markets can potentially reduce portfolio risk as well as the cost of funds. Domestic and foreign interest rates and stock returns generally do not move together perfectly over time. This means that the risks from mismatching one-currency positions may be offset by potential gains from asset-liability portfolio diversification.

Each domestic and foreign nominal interest rate consists of two components. The first component is the **real interest rate,** which reflects a given currency's real demand and supply for its funds. Differences in real interest rates will cause a flow of capital into those countries with the highest available *real* rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to the currencies of countries whose available real rate of return is low.

The second component is the **expected inflation rate,** which reflects the amount of compensation required by investors to offset the expected erosion of real value over time due to inflation. Differences in inflation rates will cause the residents of the country with the highest inflation rate to demand more imported (cheaper) goods. For example, if prices in the United States are rising twice as fast as in Australia, U.S. citizens will increase their

demand for Australian goods (because Australian goods are now cheaper relative to domestic goods). If a country's inflation rate is higher than its trading partners', the demand for the country's currency will be low, and the currency will depreciate.

The **nominal interest rate**, *r*, is the compounded sum of the real interest rate, *real r*, and the expected rate of inflation, *E*(i), over an estimation horizon.

$$\text{exact methodology: } (1 + \text{r}) = (1 + \text{real r})[1 + \text{E(i)}]$$

$$\text{linear approximation: } \text{r} \approx \text{real} + \text{E(i)}$$

Key Concepts

LO 45.1

Net exposure in a foreign currency measures the extent to which a bank is net long or net short a foreign currency. A financial institution's net currency exposure is calculated as:

net currency exposure = (currency assets – currency liabilities)
+ (currency bought – currency sold)

LO 45.2

A net long (short) position in a currency means that a bank faces the risk that the FX rate will fall (rise) in value versus the domestic currency.

LO 45.3

If a financial institution fails to maintain a balanced position, the institution will be exposed to variations in the FX rate. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

LO 45.4

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

- Enabling customers to participate in international commercial business transactions.
- Enabling customers (or the financial institution itself) to take positions in real and financial foreign investments.
- Offsetting exposure to gain currency for hedging purposes.
- Speculating on future FX rate movements.

LO 45.5

Most of the profits and losses on FX come from speculation or open position taking. A secondary source of revenue comes from market-making activities and/or agency fees. Mismatches between foreign financial assets and liabilities can create FX risk exposure.

LO 45.6

Returns for the bank's portfolio are derived from differences between income and costs. However, there is an extra dimension of return and risk from adding foreign currency assets and liabilities to a portfolio.

LO 45.7

There are two principle methods of better controlling the impact of FX exposure:

- On-balance-sheet hedging is achieved when a financial institution has a matched maturity and foreign currency balance sheet.
- Off-balance-sheet hedging occurs through the purchase of forwards for institutions that choose to remain unhedged on the balance sheet.

LO 45.8

Interest rate parity (IRP) suggests that the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. IRP can be stated using continuously compounded rates as follows:

$$\text{forward rate} = \text{spot rate} \times e^{(r_{DC} - r_{FC})T}$$

LO 45.9

Since domestic and foreign interest rates and stock returns do not usually move in perfect correlation, opportunities for potential gains from asset-liability portfolio diversification can offset currency risk.

LO 45.10

The real interest rate reflects a given currency's real demand and supply for its funds. The nominal interest rate is the compounded sum of the real interest rate and the expected rate of inflation over an estimation horizon.

Concept Checkers

1. Ion National Bank issues a 6-month, USD1 million CD at 4.0% and funds a loan in Argentine pesos (ARS) at 6.50%. The spot rate for the ARS was ARS2.27 per USD at the time of the transaction. In 6 months, the ARS will have depreciated to ARS2.30 per USD. What is the realized nominal annual spread on the loan?
   A. –1.07%.
   B. –0.19%.
   C. 0.11%.
   D. 0.13%.

2. With respect to Japanese yen (JPY), a U.S. firm has exchange-rate risk:
   A. that depends only on its net asset-liability position.
   B. if its JPY-denominated bonds have greater value than its JPY-denominated loans.
   C. only if its net JPY position is positive.
   D. whenever its total JPY assets are not equal to its total JPY liabilities.

Use the following data to answer Questions 3 through 5.

Century Bank issues USD20 million in U.S. CDs to fund its loan portfolio. The following characteristics pertain to the asset-liability position of the bank:

- A promised 1-year rate on the CDs of 7%.
- It invests 50% of its USD20 million in 1-year U.K. loans at 12% (loans made in GBP).
- The bank invests the other 50% in U.S. loans at 8% for one year.
- At the beginning of the year, the bank sells USD10 million for GBP in the spot currency markets at an exchange rate of USD1.42/GBP.
- The 1-year forward exchange rate is USD1.40/GBP.

3. If the spot foreign exchange rate does not change over the year, the USD proceeds from the U.K. investment will be:
   A. USD7,040,000.
   B. USD7,890,000.
   C. USD11,200,000.
   D. USD12,000,000.

4. If the exchange rate falls to USD1.38/GBP, what is the weighted return on the bank's asset portfolio?
   A. 1.41%.
   B. 2.82%.
   C. 5.41%.
   D. 8.42%.

5. If the bank hedges its GBP loan in the forward market, what is the return on the bank's loan portfolio?
   A. 8.37%.
   B. 9.21%.
   C. 9.79%.
   D. 10.11%.

Concept Checker Answers

1. **B** USD1 M × ARS2.27 × 1.0325 = ARS2,343,775 / 2.30 = USD1,019,033 – (USD1 M × 1.02) = –\$967.40; –967.40/1 M = –0.0009674 × 2 = –0.19%

2. **D** A firm's exchange-rate risk depends on its net asset-liability exposure and on the volatility of the exchange rate with the JPY. Bonds and loans are only part of the whole JPY-denominated portfolio; forward contracts and currency holdings must be included to calculate the net asset-liability exposure. Either a positive or negative imbalance between JPY-denominated assets and liabilities will expose the firm to exchange rate risk.

3. **C** USD1.42/GBP = USD10,000,000 / 1.42 = GBP7,042,254 (1.12) = GBP7,887,324 × 1.42 = USD11,200,000

4. **D** USD10,000,000 × 1 / 1.42 = GBP7,042,254

   GBP7,042,254 × 1.12 × USD1.38/GBP = USD10,884,507

   (USD10,884,507 – USD10,000,000) / 10,000,000 = 0.08845 = 8.845%

   (0.5)(0.08) + (0.5)(0.08845) = 8.42%

5. **B** USD 10,000,000 × 1 / 1.42 = GBP7,042,254 × 1.12 × USD 1.40/GBP = USD 11,042,254; (11,042,254 – 10,000,000) / 10,000,000 = 10.42%; (0.5)(0.08) + (0.5)(0.1042) = 9.21%

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Corporate Bonds

Topic 46

Exam Focus

The term "bond" refers to a variety of assets which offer a wide range of interest rate payments from fixed cash payments, to accruals without cash, to payments in the form of additional securities. In this topic, we will provide an overview of major fixed-income instruments and their payment structures. We will also address the impact of credit risk and event risk on bond ratings and features. For the exam, be familiar with the types of bonds discussed and the methods for retiring bonds. Also, know the terminology associated with high-yield issues.

Bond Indenture and Role of Corporate Trustee

LO 46.1: Describe a bond indenture and explain the role of the corporate trustee in a bond indenture.

The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders. Banks or trust companies most often serve as corporate trustees, and the position requires that they act in a fiduciary capacity on behalf of the bondholders. The trustee would authenticate the issue, which includes keeping track of the amount of bonds issued and making sure the number does not exceed the limit specified in the indenture. The trustee would monitor the corporation's activities to make sure the issuer abides by the indenture's covenants (e.g., maintaining key ratios below a given number).

All corporate bond offerings over $5 million and sold in interstate commerce must have a corporate trustee as set forth in the **Trust Indenture Act**. The corporate trustees must be competent and financially responsible and should also not have any conflicts of interest, (e.g., being a creditor of the issuer). The indenture would specify how the trustee would make reports to bondholders and what to do if the issuer fails to pay interest or principal. As mentioned earlier, the basic goal of the trustee is to protect the rights of bondholders.

Maturity Date

LO 46.2: Explain a bond's maturity date and how it impacts bond retirements.

The maturity date of a bond is when the bond issuer's obligations are fulfilled. At maturity, the issuer pays the principal and any accrued interest or premium. The contract, as set forth by the indenture, may terminate prior to the maturity date if, for example, the corporation chooses to retire the bonds early. The longer the maturity of the bond, the more time a company has to retire the bond issue.

Interest Payment Classifications

LO 46.3: Describe the main types of interest payment classifications.

LO 46.4: Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk.

The main types of bond interest payment classifications are: straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. The interest rate on a bond is often called the **coupon**. However, bonds today technically no longer have coupons attached directly to them. Now, bonds are registered and represented by a certificate, or they are kept in book-entry form where one master or global certificate is issued and held by a central securities depository that issues receipts. This method is considered a safer way to make payments.

Straight-coupon bonds, also called fixed-rate bonds, have a fixed interest rate set for the entire life of the issue. In the United States, fixed-rate bonds typically pay interest every six months. In Europe and some other countries, bonds make annual interest rate payments. A bond issued in the United States with an 8% interest rate and a $1,000 par value on March 1, 2009 will pay $40 of interest each September 1 and each March 1 until its maturity date or until the bond is retired, at which time the issuer would pay both the final interest payment and the $1,000 principal back to the bondholder.

In addition to just paying a fixed dollar interest, bonds in the United States have been issued that pay in foreign currency. Two other variations are a participating bond and an income bond. **Participating bonds** pay at least the specified interest rate but may pay more if the company's profits increase. **Income bonds** pay at most the specified interest, but they may pay less if the company's income is not sufficient. In both cases, the conditions for paying more or less than the specified coupon would be set forth in the indenture.

Floating-rate bonds are also known as variable rate bonds. The interest paid is generally linked to some widely used reference rate such as LIBOR or the Federal Funds rate.

Zero-coupon bonds pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the full face value at maturity. Variations of the zero-coupon bond include the **deferred-interest bond** (DIB) and the **payment-in-kind bond** (PIK). The DIB will not pay cash interest for some number of years early in the life of the bond. That period is the deferred-interest period. During this period, cash interest accrues and is then paid semiannually until maturity or when redeemed. PIK bonds pay interest with additional bonds for the initial period, and then cash interest after that period ends.

Most zero-coupons issued today share a host of other features such as being convertible, callable, and putable. A zero-coupon bond's interest rate is determined by the original-issue discount (OID):

original-issue discount (OID) = face value – offering price

The value of the bond grows each year and thus pays implicit interest, which is a function of the OID and the term-to-maturity. In other words, the rate of return depends on the amount of the discount and the period over which it grows.

One advantage of zero-coupon bonds is zero **reinvestment risk.** The bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. A disadvantage is that the bondholder must pay taxes each year on the accrued interest even though no cash is received from the bond issuer.

If the issuer goes into bankruptcy prior to the maturity of a zero-coupon bond, the bondholders are only entitled to the accrued interest up to that date and not the full face value of the bond. In other words, the zero-coupon bond creditor can only claim the original offering price plus accrued and unpaid interest up to the date of the bankruptcy filing. The bond issuer faces a huge liability with a zero-coupon bond because of the large balloon payment at maturity.

Bond Types

LO 46.5: Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds.

Corporate bonds can have a security, such as real property, underlying the issue. Those who own mortgage bonds have a first-mortgage lien on the properties of the issuer. This security allows the issuer to pay a lower rate of return than it would have to pay on unsecured bonds, which are known as debentures. The lien gives the bondholders the right to sell the mortgaged property to satisfy unpaid obligations to bondholders. In practice, this right is usually used for bargaining purposes only, and the bankruptcy takes the form of reorganization as opposed to liquidation.

Mortgage bonds can be issued in a series in a blanket arrangement. In this case, one group of bonds is issued under the mortgage, and then others are issued later. When earlier issues mature, additional bonds are then issued in their place.

Collateral trust bonds are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property. The issuers are holding companies, and the collateral consists of claims on their subsidiaries.

The trustee holds the collateral for the benefit of the bondholders; however, the issuer retains voting rights for stock used as collateral, so they retain control over their subsidiaries. The indenture may have provisions covering what to do if the value of the collateral falls below the value of the loan. If the collateral falls in value, the issuer may have to contribute additional securities to back the bonds. The issuer may be able to withdraw collateral if the value rises in order to exceed the loan value. Like mortgage bonds, collateral trust bonds may be issued in series.

Equipment trust certificates (ETCs) are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs. The payments to the creditors are called dividends. The trustee pays for the equipment with the money raised from the issuance of the ETCs, usually about 80% of the value of the equipment, and what is effectively a down payment from the user of the equipment. This provides more security to the creditors than that of a mortgage bond. It is especially attractive if the equipment is standardized, as in the case of railroad cars, which provides for easy sale or lease of the equipment in the case the user of the equipment defaults. ETCs are generally considered the most secure type of bond since the underlying assets are actually owned by the trustee and rented to the borrower.

As noted earlier, **debentures** are unsecured bonds (i.e., they do not have any assets underlying the issue). Most corporate bonds are debentures and usually pay a higher interest rate for that reason. However, if the company is highly rated and has not issued any secured bonds, then debentures are almost the equivalent of mortgage bonds in that they have a claim on all the assets of the issuer along with the general creditors. If the issuer has issued secured debt along with debentures, the debenture holders have a claim on the assets that are not backing the secured debt. Typically the issuer is restricted to one issue of debentures if there is already secured debt. If there is no secured debt, and the company issues debentures, there is often a negative-pledge clause that says that the debentures will be secured equally with any secured bonds that may be issued in the future.

Subordinated debenture bonds have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them. This means that the issuer has to offer a higher interest rate on the subordinated debentures.

Issuers may choose to issue **convertible debentures**, which give the bondholder the right to convert the bond into common stock. This feature will lower the interest rate paid. The cost to the issuer, however, is the possibility of increased dilution of the stock. A variation of convertible debentures is **exchangeable debentures** that are convertible into the common stock of a corporation other than that of the issuer.

Bonds issued by one company may also be guaranteed by other companies. These bonds are known as **guaranteed bonds**. A guarantee does not ensure that the issue will be free of default risk since the risk will depend on the ability of the guarantor(s) to satisfy all obligations.

Methods for Retiring Bonds

LO 46.6: Describe the mechanisms by which corporate bonds can be retired before maturity.

There are a variety of methods for retiring debt, and some are included in the bond's indenture while others are not included. The indenture would include the call and refunding provisions, sinking funds, maintenance and replacement funds, and redemption through sale of assets. The indenture would not include fixed-spread tender offers.

Call and refunding provisions are essentially call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part prior to maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon. Other reasons for exercising these options are to eliminate restrictive covenants, alter capital structure, increase shareholder value, or improve financial/managerial flexibility. A **call provision** can either be a fixed-price call or a make-whole call.

- **Fixed-price call.** The firm can call back the bonds at specific prices that can vary over the life of the bonds as specified in the indenture. They generally start out high and decline toward par. Also, for most bonds, the bonds are not callable during the first few years of the issue's life.
- **Make-whole call.** In this case, market rates determine the call price, which is the present value of the bond's remaining cash flows subject to a floor price equal to par value. A discount rate based on the yield of comparable-maturity Treasury securities (usually the rate plus a premium) determines the present value and the bond's price. The redemption price is the greater of that present value or the par value plus accrued interest.

A **sinking fund provision** generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market. The purchase of some sufficient amount of equipment in excess of the value of the amount of the bonds to be retired is another action that may satisfy a sinking fund provision.

The lottery approach to satisfying the sinking fund is very similar to a call provision in that the bondholders must sell back their bonds at a specified price. Unlike the call provision, there may be advantages to the bondholders. First, the retirement of bonds improves the financial health of the firm. Second, the redemption price may exceed the market price. However, the indenture may give the issuer some flexibility as to how much of the bonds to call back each period, which would give the firm some latitude to call back more bonds when the market conditions are favorable to do so. One example is an accelerated sinking-fund provision, which allows the firm to call back more bonds in early years, which the firm would do if interest rates fall in those early years.

A **maintenance and replacement fund** (M&R) has the same goal as a sinking fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the M&R provision is more complex since it requires valuation formulas for the underlying assets. The main point is that the provision specifies that the fund must keep up the value of the underlying assets much like a home mortgage specifies the home buyer must keep up the value of the home. One way to satisfy the provision is

to acquire sufficient cash to maintain the health of the firm. That cash can then be used to retire debt.

Tender offers are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. The goal is to eliminate restrictive covenants or to use excess cash. If the first tender offer price does not get sufficient interest, the firm can increase its offer price. Firms can also announce that they will buy back bonds based on the price as determined by a certain market interest rate (e.g., the yield to maturity on a comparable-maturity Treasury plus a spread). This lowers interest rate risk for both the bondholders and the bond issuer.

As a final note, the issuing firm may be able to call back bonds if it is necessary to sell assets associated with the bond issue. For example, if the government requires a firm to sell property, but that property is being used as collateral for the bonds, the firm would sell the property and call back the bonds.

Credit Risk

LO 46.7: Differentiate between credit default risk and credit spread risk.

Credit risk includes credit default risk and credit spread risk. **Credit default risk** is the uncertainty concerning the issuer making timely payments of interest and principal as prescribed by the bond's indenture. The most widely-used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States. The agencies assign a symbol associated with the rating (e.g., AAA or Aaa for the corporate debt with the least credit default risk). The rating can be interpreted as a probability of default within some time period.

Professor's Note: We will discuss credit ratings in more detail in Topic 48.

Credit spread risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread.** It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk.

The risk here is from possible changes in this spread from changes in investor risk aversion, which will change the value of the associated bond. Other factors affecting credit spreads are macroeconomic forces such as the level and slope of the Treasury yield curve, the business cycle, and issue-specific factors such as the corporation's financial position and the future prospects of the firm and its industry.

A method commonly used to evaluate credit spread risk is **spread duration**. The duration of the spread is the approximate percentage change in a bond's price for a 100 basis point change in the credit spread assuming that the Treasury rate is constant. If a bond has a spread duration of 4, for example, a 50 basis point change in the spread will change the value of the bond by 2%.

Event Risk

LO 46.8: Describe event risk and explain what may cause it in corporate bonds.

Event risk addresses the adverse consequences from possible events such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases, which may escape being included in the indenture. Such events can drastically change the firm's capital structure and reduce the creditworthiness of the bonds and their value. In order to protect shareholders, a company may include in the indenture a **poison put**, which can require the company to repurchase the debt at or above par value in the event of a takeover not approved by the board of directors (i.e., a hostile takeover). The purpose of this feature is to protect bondholders, but its effectiveness toward this goal can be misleading in that the acquiring firm may offer a sufficiently high price for the stock so that the hostile takeover becomes friendly. As a result, the poison puts would not be exercised.

Investors can lobby for clauses in the indenture to activate a put option for a variety of reasons including a change in the bond's rating. Some of the debt rating services issue commentary on the indenture's protective features, which could include the possibility of the firm being able to circumvent the features through careful legal moves (e.g., turning a hostile takeover into a friendly takeover). It should be noted that event risk can change on a market level. During times of increased merger activity, for example, the event risk increases for most bonds.

High-Yield Bonds

LO 46.9: Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds.

High-yield bonds (a.k.a. junk bonds) are those bonds rated below investment grade by ratings agencies. This includes a broad range of ratings below the cutoff, (e.g., BB to default). **Businessman's risk** refers to bonds with a rating at the bottom rung of the investment-grade category (Baa and BBB) or at the top end of the speculative-grade category (Ba and BB). Over long periods of time, high-yield bonds should offer higher average returns. However, over shorter periods, the returns will be volatile where large losses are possible.

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment grade rating. Such issuers include young and growing companies that do not have strong financial statements but who have promising prospects. Firms may issue such bonds to raise venture capital, and their prospects are tied to a particular project or story, which gives them the name "story bonds."

Established firms that have had a deteriorating financial situation may need to raise debt capital as well, and they would issue bonds that reflect their situation. Also, an established firm who already has unsecured debt issued with an investment-grade credit rating may be able to issue subordinated debt, but that debt would be non-investment grade.

Fallen angels are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade. If the issuers are in or near bankruptcy, they are often called "special situations," which could either pay off if the company recovers or lead to big losses.

Restructurings and **leveraged buyouts** may increase the credit risk of a company to the point where the bonds become non-investment grade. The new management may pay high dividends, deplete the acquired firm's cash, and lower the rating of the existing bonds. In this process, the firm may issue non-investment grade debt to pay off the bridge loans taken to finance the acquisition.

High-yield bonds can have several types of coupon structures. There are **reset bonds**, where designated investment banks periodically reset the coupon to reflect market rates and the creditworthiness of the issuer. There are also **deferred-coupon structures**, which include three types: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds. Deferred-interest bonds sell at a deep discount and do not pay interest in the early years of the issue, say, for three to seven years. Step-up bonds pay a low coupon in the early years and then a higher coupon in later years. Payment-in-kind bonds allow the issuer to pay interest in the form of additional bonds over the initial period.

Default Rate

LO 46.10: Define and differentiate between an issuer default rate and a dollar default rate.

A default occurs if there are any missed or delayed disbursements of interest and/or principal. It has been proven that lower credit ratings indicate a higher probability of default, but there are two ways to measure default: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. For each approach in measuring default rates, there are different formulas, which can lead to researchers reporting different default rates for the same data set.

The **issuer default rate** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It is only a proportion of the number of issuers who do fulfill their obligations and does not include a measure of the dollar amount involved.

The **dollar default rate** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year. Over a multi-year period, often-used measures are ratios of cumulative dollar value of all defaulted bonds

divided by some weighted-average measure of all bonds issued. One such measure attempts to weight the bonds outstanding by the number of years they are in the market:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Another measure simply takes a raw total as shown in the following equation:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$$

Recovery Rate

LO 46.11: Define recovery rates and describe the relationship between recovery rates and seniority.

The recovery rate is the amount received as a proportion of the total obligation after a bond defaults. Measuring this can be complicated because the value of the total obligation requires computing the present value of the remaining cash flows at the time of the default. Furthermore, some of the amount that the investor recovers may be in the form of securities (e.g., stock in the company). A study by Moody's estimated that the recovery rate for bonds has been about 38%. Bonds with higher seniority will obviously have higher recovery rates.

KEY CONCEPTS

LO 46.1

A bond indenture sets forth the obligations of the issuer. The trustee interprets the legal language of the indenture and works to make sure the issuer fulfills obligations to bondholders.

LO 46.2

The bond issuer's obligations are fulfilled on the maturity date or before. Bonds can be retired before that date.

LO 46.3

The main types of interest payment classifications are straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. Straight-coupon bonds pay a fixed cash coupon periodically. Floating-rate bonds pay a cash amount that varies with market rates. Zero-coupon bonds increase in value over the life of the issue.

There are many variations of the main types of bond structures. For example, deferred-interest bonds are a mix of zero-coupon and coupon bonds in that they do not pay cash interest in early years and pay a cash coupon in later years. Some bonds have principal in one currency and pay coupons in another currency.

LO 46.4

Zero-coupon bonds have low reinvestment risk. The interest is based on the time-to-maturity at issuance and the original-issue discount, which is the difference between the face value and the offering price. In the case of bankruptcy, the bondholder has a claim only equal to the issue price plus accrued interest to that date, and not the full face value.

LO 46.5

The holder of a mortgage bond has the first lien on real property owned by the issuer.

Collateral trust bonds are backed by stocks and bonds that represent claims against the subsidiaries of the issuer. The collateral is also called personal property.

Equipment trust certificates are a form of mortgage bond where the trustee actually owns the property and rents it to the bond issuer. The property is often in the form of standardized equipment (e.g., rail cars) that is easily sold.

Debentures are unsecured debt. Owners of debentures have a claim on the company's assets not backing outstanding secured debt.

LO 46.6

Call provisions allow the firm to retire debt early at a given price. Sinking-fund provisions require the firm to buy back portions of debt each year. Call provisions are generally considered detrimental to bondholders, but sinking-fund provisions may be beneficial to bondholders.

A maintenance and replacement fund helps maintain the financial health of the firm. Cash in the fund can be used to retire debt.

Bond issuers can retire debt through a tender offer. The offer price may either be a fixed price or a price that varies with a market rate such as that on comparable Treasury securities.

LO 46.7

Credit default risk is the possibility that the issuer does not make the payments specified in the indenture. Credit spread risk is the price risk from changes in the spread of a bond's interest rate over the corresponding Treasury rate.

LO 46.8

Event risk is the possibility that a merger, restructuring, acquisition, et cetera, increases the risk of the bond by changing the ability of the firm to pay off the bonds. The indenture can try to address some of these events, but some can be omitted and lawyers can find loopholes around those included.

LO 46.9

High-yield bonds can either be issued by growing, risky firms or established firms with senior debt outstanding. High-yield bonds may also be fallen angels (i.e., one-time investment grade bonds).

High-yield bonds may have coupon structures which allow the firm to conserve cash in early years, such as: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds.

LO 46.10

The issuer default rate is a proportion based on the number of issues that default as a proportion of all issues. The dollar default rate estimates the dollar amount of defaulted bonds compared to the dollar amount of the corresponding population of bonds outstanding.

LO 46.11

In the event of default, the recovery rate refers to the amount a bondholder receives as a proportion of the amount owed. Bonds with higher seniority usually have higher recovery rates.

Concept Checkers

1. Which of the following responsibilities is least likely to be part of the role of a corporate trustee in a bond issue?
   A. Interpret the language of the indenture.
   B. Determine the interest rate on a reset bond.
   C. Keep track of the amount of bonds issued by the corporation.
   D. Monitor the corporation's activities to make sure the corporation abides by the indenture's covenants.

2. In bankruptcy, the holder of a zero-coupon bond obligation of the bankrupt corporation would have a claim equal to:
   A. the face value of the bond.
   B. the issuing price of the bond only.
   C. the issuing price plus accrued interest.
   D. nothing, since zeros are always unsecured.

3. All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
   A. Equipment trust certificates.
   B. Mortgage bonds.
   C. Junior debentures.
   D. Senior debentures.

4. Which of the following methods for retiring bonds before maturity is generally considered the most detrimental for the bondholders?
   A. Tender offers.
   B. Call provision.
   C. Sinking fund provision.
   D. Maintenance and replacement funds.

5. With respect to default risk and credit spread risk, the ratings of bond-rating agencies such as Moody's provide information concerning:
   A. default risk only.
   B. credit spread risk only.
   C. both default risk and credit spread risk.
   D. neither default risk nor credit spread risk.

Concept Checker Answers

1. **B** Investment banks other than the trustee set the rate on a reset bond.

2. **C** The claim equals the value at that point in time as implied by the issuing price, the original-issue discount, and accrued interest.

3. **A** ETCs, or equipment trust certificates, are generally the most secure because the underlying assets are actually owned by the trustee and rented to the borrower. Also, the assets are usually standardized for easy resale.

4. **B** The call provision gives the issuer the right to purchase the bonds at a given price, which the issuer would not do unless that price was below the market price. Sinking fund provisions can benefit bondholders because the issuer is obligated to purchase bonds, which improves the creditworthiness of the issue, and the issuer may have to do so at a price higher than the market price. There are no features in M&R funds or tender offers that would be detrimental to bondholders.

5. **A** Bond rating agencies issue ratings based on the default risk of the issue. Credit spread risk is determined by spread duration.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

Mortgages and Mortgage-Backed Securities

Topic 47

Exam Focus

Mortgage-backed securities (MBSs) are debt securities backed by a pool of residential loans, which serve to transform mortgages from an illiquid asset into a liquid asset. Because the underlying mortgages can be prepaid, prepayment risk is a major concern for MBS investors. Monte Carlo simulation is the most common methodology used for valuing MBSs because it is able to account for prepayment risk. Alternate interest rate paths are assumed in the model to generate an option-adjusted spread (OAS). For the exam, be able to calculate the payments for a fixed-rate, level paying mortgage. Also, be familiar with the factors that affect prepayment rates and how to measure prepayment speeds with a conditional prepayment rate (CPR). Finally, be prepared to discuss the steps involved in valuing an MBS using the Monte Carlo methodology and understand the advantages and disadvantages of using an OAS.

Residential Mortgage Products

LO 47.1: Describe the various types of residential mortgage products.

A **mortgage** is a loan that is collateralized with a specific piece of real property. Before the 1970s, mortgages existed solely in the **primary market** where banks that issued the mortgage loans collected all interest and principal payments from the borrower. Within the past few decades, it is more common for mortgage lenders to sell the loans in the **secondary market** through a process known as **securitization**. The secondary market has allowed more banks to issue mortgage loans.

In the secondary market, mortgages are pooled together and packaged to investors in the form of a **mortgage-backed security** (MBS). The payments of an MBS can follow a **pass-through structure** where the interest and principal collected from the borrower pass through the banks and ultimately end up with the MBS investor. Because default risk is present in mortgage lending, banks will often guarantee the borrower's payments when mortgages are securitized.

Lien Status

Whether the mortgage is a first lien, a second lien, or a subsequent lien will greatly impact the lender's ability to recover the balance owed in the event of default. For example, a first lien would give the lender the first right to receive proceeds on liquidation, so from a seniority perspective, a first lien is more desirable than a second lien.

Original Loan Term

Mortgage terms of 10 to 30 years are common, with the most popular being 30 years (long term). However, medium terms in the 10- to 20-year range are starting to become more common, given the desire of many individuals to pay off their mortgages as soon as possible.

Credit Classification

Classifying loans between prime and subprime is determined mainly by credit score (i.e., Fair Isaac Corporation or FICO model).

Prime (A-grade) loans constitute most of the outstanding loans. They have low rates of delinquency and default as a result of low **loan-to-value (LTV) ratios** (i.e., far less than 95%), borrowers with stable and sufficient income (i.e., *front income ratio* of no more than 28% of monthly income to service payments relating to the home and *back income ratio* of no more than 36% for those payments plus other debt payments), and a strong history of repayments (e.g., FICO score of 660 or greater). Home payments include interest, principal, property taxes, and homeowners insurance.

Subprime (B-grade) loans have higher rates of delinquency and default compared to prime loans. They could be associated with high LTV ratios (i.e., 95% or above), borrowers with lower income levels, and borrowers with marginal or poor credit histories (e.g., FICO score below 660). High LTV ratios suggest a higher risk of default. Upon issuance, subprime loans are carefully scrutinized by the servicer to ensure timely payments.

Alternative-A loans are the loans in between prime and subprime. Although they are essentially prime loans, certain characteristics of Alternative-A loans make them riskier than prime loans. For example, the loan value may be unusually high, the LTV ratio may be high, or there may be less documentation available (e.g., income verification, down payment source).

Interest Rate Type

Fixed-rate mortgages have a set rate of interest for the term of the mortgage. Payments are constant for the term and consist of blended amounts of interest and principal.

Adjustable-rate mortgages (ARMs) have rate changes throughout the term of the mortgage. The rate is usually based on a base rate (e.g., prime rate, LIBOR) plus a spread. Rates can usually change on a monthly, semiannual, or annual basis. The risk of default is high, especially if there are large rate increases after the first year, thereby significantly increasing the total payment amount (due to the increase in interest).

Prepayments and Prepayment Penalties

Prepayments reduce the mortgage balance and amortization period. They can occur because of the following reasons:

- Home is sold, which requires the mortgage balance to be paid off.
- Refinancing due to lower rates or more attractive loan features elsewhere.
- Partial prepayments by the borrower during the term.

To counteract the negative effects of prepayments, many loans contain prepayment penalties. They are amounts payable to the servicer for prepayments within a certain time and/or over a certain amount. Soft penalties are those that may be waived on the sale of the home; hard penalties may not be waived.

Credit Guarantees

The ability to create mortgage-backed securities requires loans that have credit guarantees.

Government loans are those that are backed by federal government agencies (e.g., Government National Mortgage Association or GNMA).

Conventional loans could be securitized by either government-sponsored enterprises (GSEs): Federal Home Loan Mortgage Corporation (FHLMC) or Federal National Mortgage Association (FNMA). For a guarantee fee, these GSEs will guarantee payment of principal and interest to the investors.

Agency (or conforming) MBSs are those that are guaranteed by any of three government-sponsored entities (GSEs): GNMA, FNMA, and FHLMC. Most of the MBSs are issued by these GSEs.

Also known as **private label** securitizations, non-agency (or non-conforming) MBSs grew along with U.S. home prices over time up to the 2007 credit crisis. The GSEs have restrictions on what mortgages they can guarantee/securitize [e.g., dollar value limit, loan-to-value (LTV) ratio limit], which opened up the private label market for those participants willing to take on the risks inherent in nonconventional loans—**jumbo loans** (mortgage principal balance over the limit) and/or loans with high LTVs. The rising prices of the underlying homes held as collateral provided some risk mitigation. Unfortunately, the falling prices of homes and the credit crisis beginning in 2007 caused a significant drop in MBS issuances in the non-agency segment because they did not have government guarantees.

With agency MBSs, the investor bears no credit risk because the GSEs have been paid a fee to guarantee the underlying mortgages. If there is a default with a mortgage, the GSE will pay the outstanding balance to the investors. With a non-agency MBS, there is some credit risk but that is mitigated through the process of subordination.

Fixed-Rate, Level-Payment Mortgages

LO 47.2: Calculate a fixed rate mortgage payment, and its principal and interest components.

A **conventional mortgage** is the most common residential mortgage. The loan is based on the creditworthiness of the borrower and is collateralized by the residential real estate that it is used to purchase. If a borrower's credit quality is questionable or the borrower is lacking a sufficient down payment, the mortgage lender may require mortgage insurance to guarantee the loan. Mortgage insurance is made available by both government agencies and private insurers. The cost of the insurance is borne by the borrower and effectively raises the interest rate on the mortgage loan.

There are a wide variety of mortgage designs that specify the rates, terms, amortization, and repayment methods. All of the concepts associated with risk analysis and valuation, however, can be understood through an examination of **fixed-rate, level payment, fully amortized mortgage loans**. This common type of mortgage loan requires equal payments (usually monthly) over the life of the mortgage. Each of these payments consists of an interest component and a principal component.

There are four important features of fixed-rate, level payment, fully amortized mortgage loans to remember when we move on to mortgage-backed securities (MBS):

1. The amount of the principal payment increases as time passes.

2. The amount of interest decreases as time passes.

3. The servicing fee also declines as time passes.

4. The ability of the borrower to repay results in **prepayment risk**. Prepayments and curtailments reduce the amount of interest the lender receives over the life of the mortgage and cause the principal to be repaid sooner.

Example: Calculating a mortgage payment

Consider a 30-year, $500,000 level payment, fully amortized mortgage with a fixed rate of 12%. **Calculate** the monthly payment and **prepare** an amortization schedule for the first three months.

Answer:

The monthly payment is \$5,143.06:

N = 360; I/Y = 1.0 (12/12); PV = –500,000; FV = 0; CPT → PMT = 5,143.06

With reference to the partial amortization schedule in the figure below, the portion of the first payment that represents interest is \$5,000.00 (0.01 × \$500,000). The remainder of the payment, \$143.06 (\$5,143.06 – \$5,000.00), goes toward the reduction of principal. The portion of the second payment that represents interest is \$4,998.57 (0.01 × \$499,856.94). The remaining \$144.49 (\$5,143.06 – \$4,998.57) goes toward the further reduction of principal.

Monthly Amortization Schedule for a 30-Year, \$500,000 Mortgage Loan at 12%

Payment Number	*Initial Principal*	*Monthly Payment*	*Interest Component*	*Reduction of Principal*	*Outstanding Principal*
1	\$500,000.00	\$5,143.06	\$5,000.00	\$143.06	\$499,856.94
2	499,856.94	5,143.06	4,998.57	144.49	499,712.45
3	499,712.45	5,143.06	4,997.12	145.94	499,566.51

Notice that the monthly interest charge is based on the beginning-of-period outstanding principal. As time passes, the proportion of the monthly payment that represents interest decreases, and, because the payment is level, the proportion that goes toward the repayment of principal increases. This process continues until the outstanding principal reaches zero and the loan is paid in full.

The incremental reduction of outstanding principal is referred to as scheduled amortization (or scheduled principal repayment). The previous figure is a portion of what is commonly called an **amortization schedule**. Amortization schedules are easily constructed using an electronic spreadsheet.

The collection of payments and all of the other administrative activities associated with mortgage loans are paid for via a servicing fee, also known as the servicing spread, because it is usually built into the mortgage rate.

For example, if the mortgage rate is 10.5% and the servicing fee is 35 basis points, the provider of the mortgage funds will receive 10.15%. This amount is called the net interest or net coupon. The dollar amount of the servicing fee is based on the outstanding loan balance; thus, it declines as the mortgage is amortized. Keep in mind that the reduction in principal associated with each payment is based on the mortgage rate and is unaffected by the servicing fee.

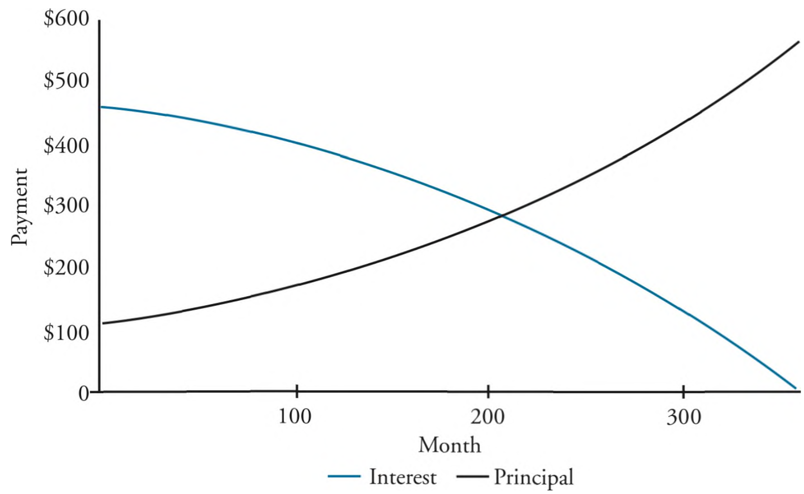
Allocation Between Principal and Interest

Fully amortizing fixed-rate mortgage:

- The mortgage payment consists primarily of interest in the early years.
- Interest is calculated on a declining principal balance so the interest payable will gradually decrease over time. As a result, more of the fixed mortgage payment will be applied toward reducing the principal amount.
- The crossover point is the point in the mortgage where principal and interest allocation amounts are the same. After that point, relatively more amounts will be allocated to principal.
- Mortgages with shorter amortization periods result in less interest paid and more of the payment applied toward reducing the principal balance sooner. In other words, equity buildup occurs at a quicker rate when the amortization period is shorter.

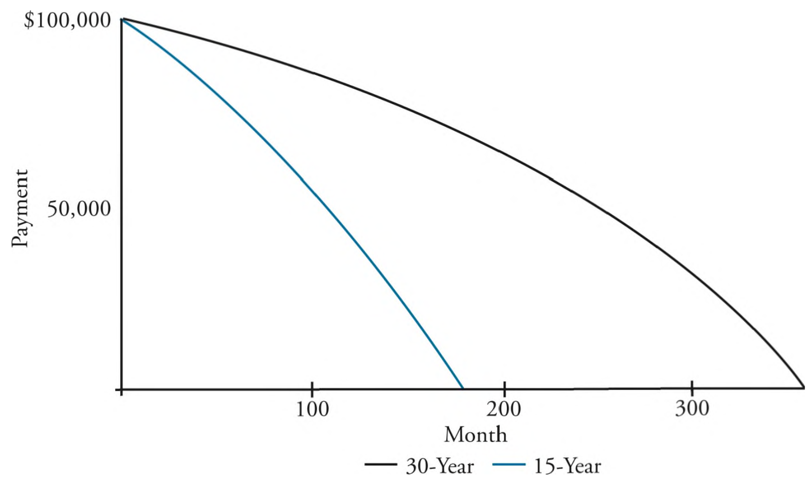
Figure 1 illustrates the relationship of interest and principal over the term of the loan.

Figure 1: Interest and Principal Over Time



As indicated previously, the reason principal payments will increase over time is due to the reduction in the outstanding loan balance. Figure 2 illustrates the relationship between loan balance and time for a $100,000 loan.

Figure 2: Loan Balance Over Time



Prepayment

LO 47.3: Describe the mortgage prepayment option and the factors that influence prepayments.

In the previous example, it was assumed that the borrower paid the exact amount of the monthly payment, and the interest and principal followed the amortization schedule. However, it is possible for a borrower to pay an amount in excess of the required payment or even to pay off the loan entirely. The option to prepay a mortgage is essentially a call option for the borrower. The borrower is in a position that is very similar to the issuer of a callable bond. A prepayment will effectively free the borrower of the mortgage obligation.

Mortgage Prepayment Option

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments (where permitted) and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate. For the lender, prepayments represent a loss for two reasons: (1) they stop receiving interest income at the high rate and (2) they have to reinvest the proceeds received from prepayment at the prevailing lower market rates. Therefore, the pricing of the initial mortgage rate should be somewhat higher to take into account the possibility of prepayment. With agency MBSs, prepayments and defaults have the same impact on investors. Prepayments result in the investors actually receiving cash from the borrowers, whereas with defaults, the borrower does not pay the outstanding mortgage balances, but the GSE does, thereby causing a prepayment.

Other Factors That Influence Prepayments

Seasonality. The summertime is a popular time for individuals to move (and mortgages must be paid out prior to the sale of a home), so it is the period of time with the greatest prepayment risk. Given some time lags, the prepayments often start to appear in the late summer and early fall.

Age of mortgage pool. Refinancing often involves penalties and administrative charges, so borrowers tend not to do so until several years into the mortgage. Also, it takes some time for borrowers to build up equity and savings to make prepayments and/or attempt to refinance. As a result, the lower the age of the mortgage pool, the less likely the risk of prepayment.

Personal. Marital breakdown, loss of employment, family emergencies, and destruction of property are commonly cited reasons for prepayments based on personal reasons. It is difficult to assess this type of prepayment risk.

Housing prices. Property value increases may spur an increase in prepayments caused by borrowers wanting to take out some of the increased equity for personal use. Property value decreases reduce the value of collateral, reduce the ability to refinance, and, therefore, decrease the risk of prepayment. The increasingly popular use of home equity lines of credit where the mortgage balance is revolving (i.e., mortgage balance can be drawn up to a certain limit and paid down to zero at any time) reduces refinancing and prepayment risk due to the nature of the loan.

Refinancing burnout. To the extent that there has been a significant amount of prepayment or refinancing activity in the mortgage pool in the past, the risk of prepayment in the future decreases. That is because presumably the only borrowers remaining in the pool are those who were unable to refinance earlier (e.g., due to poor credit history or insufficient property value), and those who did refinance have been removed from the pool already. Also, those who made only large prepayments (instead of fully refinancing) in the past would have exhausted their savings to make the prepayment and would require quite some time to do so again in the future.

Securitization

LO 47.4: Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs.

LO 47.5: Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool.

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (i.e., originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a **special purpose vehicle** (SPV), in exchange for

cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue MBSs to investors; the securities are backed by the mortgage loans as collateral.

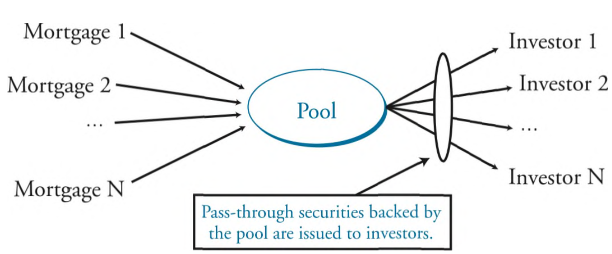
As of now, the securitization process has become a crucial part of the U.S. credit system. Financial institutions expect to originate mortgage loans and sell them through securitization. The lack of a securitization market for mortgages would lead to the downfall of mortgage lending because financial institutions would not want to retain the risks.

Pass-Through Securities

A **mortgage pass-through security** represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool and any mortgage included in the pool is referred to as a **securitized mortgage**. The mortgages in the pool have different maturities and different mortgage rates. The **weighted average maturity** (WAM) of the pool is equal to the weighted average of all mortgage ages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. The **weighted average coupon** (WAC) of the pool is the weighted average of the mortgage rates in the pool. The investment characteristics of a mortgage pass-through are a function of its cash flow features and the strength of its government guarantee.

As illustrated in Figure 3, pass-through security investors receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that **pass-through rates** (i.e., the coupon rate on the pass-through) are less than the average coupon rate of the underlying mortgages in the pool.

Figure 3: Mortgage Pass-through Cash Flow



Because pass-through securities may be traded in the secondary market, they effectively convert illiquid mortgages into liquid securities (as mentioned, this process is called **securitization**). More than one class of pass-through securities may be issued against a single mortgage pool.

The timing of the cash flows to pass-through security holders does not exactly coincide with the cash flows generated by the pool. This is due to the delay between the time the mortgage service provider receives the mortgage payments and the time the cash flows are "passed through" to the security holders.

The most important characteristic of pass-through securities is their prepayment risk; because the mortgages used as collateral for the pass-through can be prepaid, the pass-throughs themselves have significant prepayment risk.

Measuring Prepayment Speeds

Prepayments cause the timing and amount of cash flows from mortgage loans and MBSs to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. Two industry conventions have been adopted as benchmarks for prepayment rates: the **conditional prepayment rate** (CPR) and the **Public Securities Association** (PSA) prepayment benchmark.

The *CPR* is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the **single monthly mortality rate** (SMM) (also referred to as constant maturity mortality) using the following formula:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

If given the SMM rate, you can annualize the rate to solve for the CPR using the following formula:

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

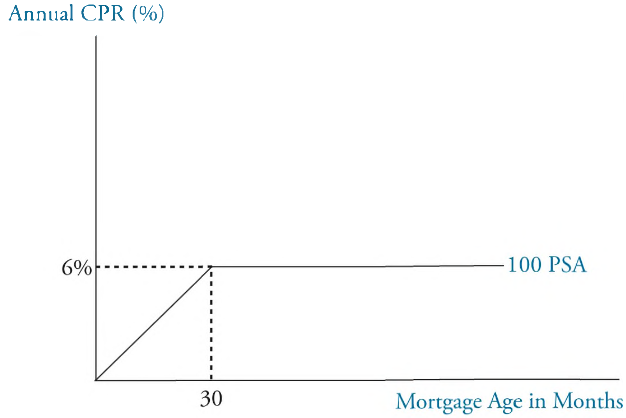
An SMM of 10% implies that 10% of a pool's beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The *PSA prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

The PSA standard benchmark is referred to as 100% PSA (or just 100 PSA). 100 PSA (see Figure 4) assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is 14(0.2%) = 2.8%.
- CPR = 6% for months 30 to 360.

Figure 4: 100 PSA



Remember that the CPRs are expressed as annual rates.

A particular pool of mortgages may exhibit prepayment rates faster or slower than 100% PSA, depending on the current level of interest rates and the coupon rate of the issue. A 50% PSA refers to one-half of the CPR prescribed by 100% PSA, and 200% PSA refers to two times the CPR called for by 100% PSA.

Example: Computing the SMM

Compute the CPR and SMM for the 5th and 25th months, assuming 100 PSA and 150 PSA.

Answer:

Assuming 100 PSA:

CPR(month 5) = $5 \times 0.2\% = 1\%$
100 PSA = $1 \times 0.01 = 0.01$
SMM = $1 - (1 - 0.01)^{1/12} = 0.000837$

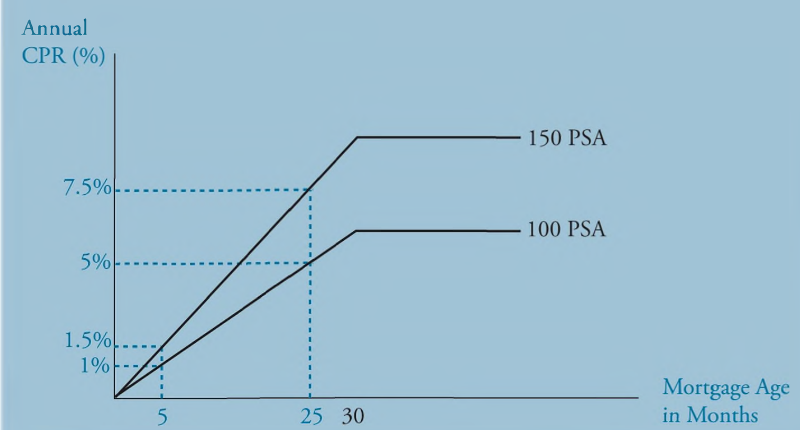
CPR (month 25) = $25 \times 0.2\% = 5\%$
100 PSA = $1 \times 0.05 = 0.05$
SMM = $1 - (1 - 0.05)^{1/12} = 0.004265$

Assuming 150 PSA:

$$\text{CPR(month 5)} = 5 \times 0.2\% = 1\%$$
$$150 \text{ PSA} = 1.5 \times 0.01 = 0.015$$
$$\text{SMM} = 1 - (1 - 0.015)^{1/12} = 0.001259$$

$$\text{CPR (month 25)} = 25 \times 0.2\% = 5\%$$
$$150 \text{ PSA} = 1.5 \times 0.05 = 0.075$$
$$\text{SMM} = 1 - (1 - 0.075)^{1/12} = 0.006476$$

Figure 5: Prepayment Speeds for 5th and 25th Months at 100 and 150 PSA



It is important for you to recognize that the nonlinear relationship between CPR and SMM implies that the SMM for 150% PSA does *not* equal 1.5 times the SMM for 100% PSA. Also, keep in mind that the PSA standard benchmark is nothing more than a market convention. It is not a model for predicting prepayment rates for MBS. In fact, empirical studies have shown that actual CPRs differ substantially from those assumed by the PSA benchmark.

Trading Pass-Through Securities

Trade settlements occur every month on a predetermined basis; delivery dates during a month are specified. In addition, prices are usually quoted for three settlements months, however, trades could be done for a longer period into the future.

Fixed rate pass-through securities (i.e., agency mortgage pools) trade in one of the following ways:

- Specified pools.
- To Be Announced (TBA).

The **specified pools** market identifies the number and balances of the pools prior to a trade. As a result, the characteristics of a given pool will influence the price of a trade. For example, high loan-balance pools, which make better use of prepayment options, trade for relatively lower prices.

The **TBA** market, which is more liquid than specified pools, involves identifying the security and establishing the price in a forward market. However, there is a pool allocation process whereby the actual pools are not revealed to the seller until immediately before settlement. The characteristics of the pools that can be used for TBA trades are regulated to ensure reasonable consistency.

Dollar Roll Transaction

LO 47.6: Describe a dollar roll transaction and how to value a dollar roll.

MBS trading requires the same securities to be priced for different settlement dates. A **dollar roll transaction** occurs when an MBS market maker buys positions for one settlement month and, at the same time, sells those same positions for another month.

How to Value a Dollar Roll

The process involves assessing the income and the expenses related over the holding period. Income is determined by coupon payments, reinvested interest, and principal payments. Expenses are determined by financing costs [i.e., repurchase (repo) market]. One could purchase the security in the earlier (front) month, hold it, and then dispose of it in the later (back) month at settlement.

The back month price of a dollar roll should take both income and expenses into account so the net cash flows are equivalent to simply purchasing the security in the back month for settlement at that time. However, empirical evidence suggests that the most likely outcome is that a price drop between the two settlement dates makes purchasing the security in the back month more attractive. In other words, purchasing a position for back month settlement results in financing at an implied repo rate lower than that of the repurchase market.

Factors that impact dollar roll valuations:

- The security's coupon, age, and WAC.
- Holding period (period between the two settlement dates).
- Assumed prepayment speed.
- Funding cost in the repo market.

Factors Causing a Dollar Roll to Trade Special

When the price difference/drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading *special*. It could be caused by:

- A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).

- An increase in the front month price (due to an increased demand in the front month for deal collateral).
- Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, thereby increasing the front month price.

Other Products

Collateralized Mortgage Obligations

All investors have varying degrees of concern about exposure to prepayment risk. Some are primarily concerned with extension risk (the increase in the expected life of a mortgage pool due to rising interest rates and lower prepayment rates), while others want to minimize exposure to contraction risk (the decrease in the expected life of a mortgage pool due to falling interest rates and higher prepayment rates). Fortunately, all of the pass-through securities issued on a pool of mortgages do not have to be the same. The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of **collateralized mortgage obligations** (CMOs).

CMOs are securities issued against pass-through securities (securities secured by other securities) for which the cash flows have been reallocated to different bond classes called *tranches.* Each tranche has a different claim against the cash flows of the mortgage pass-throughs or pool from which it was derived. Each CMO tranche represents a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

Planned Amortization Class Tranches

The most common type of CMO today is the **planned amortization class** (PAC). A PAC is a tranche that is amortized based on a sinking fund schedule that is established within a range of prepayment speeds called the *initial PAC collar* or *initial PAC bond.*

What makes a PAC bond work is that it is packaged with a *support,* or *companion,* tranche created from the original mortgage pool. Support tranches are included in a structure with PAC tranches specifically to provide prepayment protection for the PAC tranches (each tranche is, of course, priced according to the timing risk of the cash flows). If prepayment rates are faster than the upper repayment rate, the PAC tranche receives principal according to the PAC schedule, and the support tranche absorbs (i.e., receives) the excess. If prepayment speeds are below the lower repayment rate, the funds needed to keep the PAC on schedule come from the cash flows scheduled for the support tranche(s). It should be pointed out that the extent of prepayment risk protection provided by a support tranche increases as its par value increases relative to its associated PAC tranche.

There is an *inverse* relationship between the prepayment risk of PAC tranches and the prepayment risk associated with the support tranches. In other words, *the certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches.*

To understand the relatively high prepayment risk for support tranches, consider the situation in which prepayments are slower than planned. Because the PAC tranches have

priority claim against the cash flows, principal payments to the support tranches must be deferred until the PAC repayment schedule is satisfied. Thus, the average life of the support tranche is extended. Similarly, when actual prepayments come at a rate that is faster than expected, the support tranches must absorb the amount that is in excess of that required to maintain the repayment schedule for the PAC. In this case, the average life of the support tranche is contracted. If these excesses continue to occur, the support tranches will eventually be paid off and the principal will then go to the PAC holders. When this happens, the PAC is referred to as a *broken* or *busted* PAC, and any further prepayments go directly to the PAC tranche. Essentially, the PAC tranche becomes an ordinary sequential-pay structure.

Notice that the prepayment risk protection provided by the support tranches causes their average lives to extend and contract. This relationship is such that as the prepayment risk protection for a PAC tranche increases, its average life variability decreases, and the average life variability of the support tranche increases.

Strips

A distinguishing characteristic of a traditional pass-through security is that the interest and principal payments generated by the underlying mortgage pool are allocated to the bondholders on a pro rata basis. This means that each pass-through certificate holder receives the same amount of interest and the same amount of principal. *Stripped MBSs* differ in that principal and interest are not allocated on a pro rata basis. The unequal allocation of principal and interest results in a price/yield relationship that is different from that of the underlying pass-through.

The two most common types of stripped MBSs are **principal-only strips** (PO strips) and **interest-only strips** (IO strips). PO strips are a class of securities that receive only the principal payment portion of each mortgage payment, while IO strips are a class that receive only the interest component of each payment.

PO strips are sold at a considerable discount to par. The PO cash flow stream starts out small and increases with the passage of time as the principal component of the mortgage payments grows. The investment performance of a PO is extremely sensitive to prepayment rates. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher yield. Since prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. The entire par value of a PO is ultimately paid to the PO investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected.

In contrast to PO strips, an IO strip cash flow starts out big and gets smaller over time. Thus, IOs have shorter effective lives than POs.

The major risk associated with IO strips is that the value of the cash flow investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested. Why? The amount of interest produced by the pool depends on its beginning-of-month balance. If market rates fall, the mortgage pool will be paid off sooner than expected, leaving IO investors with no interest cash flow. Therefore, IO investors want prepayments to be slow.

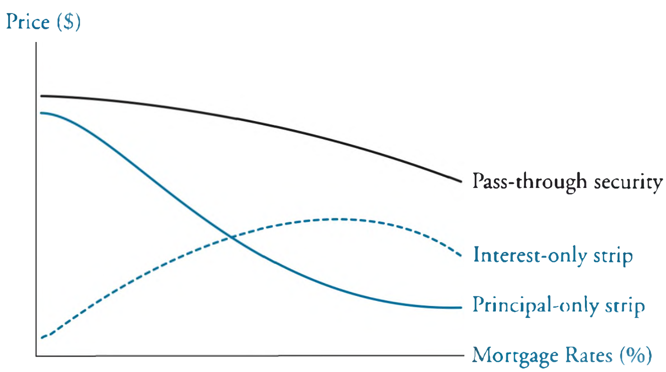
An interesting property of an IO is that its price has a tendency to move in the same direction as market rates. When market rates decline below the contract rate and prepayment rates increase, the diminished cash flow usually causes the IO price to decline, despite the fact that the cash flows are discounted at a lower rate. As interest rates rise above the contract rate, the expected cash flows improve. Even though the higher rate must be used to discount these improved cash flows, there is usually a range above the contract rate for which the price increases.

Both IOs and POs exhibit greater *price volatility* than the pass-through from which they were derived. This occurs because IO and PO returns are negatively correlated (their prices respond in opposite directions to changes in interest rates), but the combined price volatility of the two strips equals the price volatility of the pass-through.

The price/yield relationships for IO and PO securities are shown in Figure 6. Notice the following:

- The underlying pass-through security exhibits significant negative convexity.
- The PO exhibits some negative convexity at low rates.
- The IO price is positively related to mortgage rates at low current rates.
- The PO and IO prices are more volatile than the underlying pass-through.

Figure 6: Investment Characteristics of IOs and POs



Prepayment Modeling

LO 47.7: Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.

Borrowers may prepay their mortgages due to the sale or destruction of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur because the borrower has defaulted on the mortgage and the lender is forced to sell the property to cover the mortgage. Finally, many mortgages have partial prepayment privileges (curtailments) that may be used, especially when the borrower has excess cash available to do so.

Refinancing a mortgage involves using the proceeds of a new mortgage to pay off the principal from an existing mortgage. If a homeowner is holding a high interest rate mortgage and the current mortgage rates fall, the incentive to refinance is large (given that rates decline enough to cover the transaction costs of refinancing). Historically, if mortgage rates fall by more than 2%, refinancing activity increases dramatically. This is known as the *media effect* because large declines in rates will likely gain the attention of the media.

Extracting home equity is another motive for refinancing a mortgage. Given a substantial increase in property value, a borrower may take out a new mortgage with a higher balance that not only pays off the existing mortgage but also has extra cash for other purposes. Extracting home equity is also known as *cash-out refinancing*.

Incentive functions are used to model refinancing activity and are based on the term structure of mortgage rates. Past rates, also called *lagged rates*, can be included in the model to help explain refinancing behavior. Incentive functions essentially forecast the present value of any dollar gains given that a borrower will refinance.

The path that mortgage rates follow on their way to the current level affects prepayments through *refinancing burnout*. To better understand this phenomenon, consider a mortgage pool that was formed when rates were 12%, then interest rates dropped to 9%, rose to 12%, and then dropped again to 9%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 9% interest rates, most homeowners in the pool who were able to refinance would have already done so.

It is typically the case that the mortgage is due once the property is sold. This is referred to as *due on sale*. Because most borrowers sell their homes without regard for the path of mortgage rates, MBS investors will be subjected to a degree of **housing turnover** that does not correlate with the behavior of rates. One factor that slows the degree of housing turnover is known as the *lock-in effect*. This essentially means that borrowers may wish to avoid the costs of a new mortgage, which likely consists of a higher mortgage rate.

Modeling turnover typically starts with a base rate and then adjusts for seasonality (turnover is higher in the summer and lower in the winter). The turnover model may also include a *seasoning ramp*, which is partially based on improvements to creditworthiness over time, and, thus, the homeowner's increased ability to prepay the mortgage. As a group, housing turnover only accounts for 10% of overall prepayments.

When a borrower **defaults**, mortgage guarantors pay the interest and principal outstanding. These payments act as a source of prepayment. Modeling prepayments from default requires an analysis of loan-to-value (LTV) ratios and FICO scores, as well as an overall analysis of the housing market.

Partial payments by the borrower are referred to as **curtailments**. These partial payments tend to occur when a mortgage is older or has a relatively low balance. Thus, prepayment modeling due to curtailment typically takes into account the age of the mortgage.

Dynamic Valuation

LO 47.8: Describe the steps in valuing an MBS using Monte Carlo Simulation.

Any discussed earlier, mortgage borrowers have an option to prepay the underlying securities. The value of MBSs with embedded options to prepay cannot be determined using traditional option valuation techniques. Therefore, the Monte Carlo valuation methodology is used to value MBSs and other fixed-income securities with embedded options.

The **binomial model** is only applicable for securities where the decision to exercise a call option is not dependent on how interest rates evolve over time. While the binomial model is useful for callable agency debentures and corporate bonds, it is not applicable to valuing an MBS. The historical evolution of interest rates over time impacts prepayments and makes the binomial model inappropriate for MBSs.

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages. For example, if mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners who can refinance will have already done so. As mentioned earlier, this prepayment pattern is called refinancing burnout. Another problem of the path-dependency of MBSs is related to the nature of structured securities, such as collateralized mortgage obligations (CMOs). The amount a CMO tranche receives in the form of cash flows for a specific month depends on the outstanding balances of other tranches in the deal. These outstanding balances are impacted by earlier principal and interest prepayments.

The **Monte Carlo methodology** is a simulation approach for valuing MBSs. Monte Carlo is actually a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo is used to provide a probability distribution of the value of an MBS. The valuation of an MBS is influenced by future interest rates, the shape of the yield curve, future interest rate volatility, prepayment rates, default rates, and recovery rates.

Each of these variables or parameters of the Monte Carlo model could have multiple outcomes with different probabilities associated for each outcome. One valuation approach in these circumstances is the **best guess approach** where the expected value of each variable is used to estimate the value of the MBS. Unfortunately, this method is highly inaccurate. For example, suppose the probability of the best guess occurring for each variable is 70%. Then, with six different variables, the probability that the best guess MBS value will occur is only 11.8% (= 0.70^6).

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS. The other information, such as the range of possible outcomes and percentile information, is useful in gauging the value of the security.

The following steps are required to value a mortgage security using the Monte Carlo methodology:

Step 1: Simulate the interest rate path and refinancing path.
Step 2: Project cash flows for each interest rate path.
Step 3: Calculate the present value of cash flows for each interest rate path.
Step 4: Calculate the theoretical value of the mortgage security.

Step 1: Simulate the interest rate path and refinancing path.

The first step in applying the Monte Carlo approach is to estimate monthly interest rates for the entire life of the mortgage security. For example, a 30-year mortgage security would require 360 monthly interest rates. In equations to follow, the total number of months on an interest rate path will be denoted by T. Also, the total number of interest rate paths or *trials* that are simulated will be denoted by N. Random interest rate paths are generated using the term structure of interest rates and a volatility assumption. The term structure of interest rates is created using the theoretical spot rate (zero-coupon) curve for the market on the pricing date. The simulations are adjusted to ensure the average simulated price of a zero-coupon Treasury bond is equal to the actual price corresponding to the pricing date. Some models use LIBOR or swap rates instead of Treasury rates.

The dispersion of future interest rates in the simulation is determined by the volatility assumption. It is common practice to use more than one level of volatility. For example, with a short/long yield volatility approach, the volatility is specified based on maturities. One volatility number is used for shorter maturities (short yield volatility) and a second yield volatility is specified for longer maturities (long yield volatility). Short yield volatility is typically assumed to be greater than long yield volatility. When yield volatility is assumed for each maturity, it is referred to as **term structure yield volatility**.

The derivatives market is used to construct an arbitrage-free term structure of future interest rates. Short-term interest rate paths are used to discount the cash flows in Step 3 of the Monte Carlo process. These interest rate paths are also used to create the prepayment paths or *vectors*, which are cash flows for each interest rate path. The prepayment vector is computed based on refinancing rates that are available each month. The mortgagor has an incentive to refinance if the refinancing rate is low relative to the mortgagor's original coupon rate. The relationship between refinancing rates and short-term interest rates is an important assumption of the model.

Step 2: Project cash flows for each interest rate path.

Cash flows for each month on each interest rate path are equal to the scheduled principal for the mortgage pool, the net interest, and prepayments. Scheduled principal payments are simply calculated based on the projected mortgage balance from the prior month. A prepayment model is used rather than a simple prepayment rate. A prepayment rate is specified for each month on a given interest rate path, and rates for a given month across all interest rate paths are not the same. In fact, there could actually be $T \times N$ different prepayment rates.

CMO deal structures dictate how principal and interest is to be paid. Therefore, it is necessary to reverse engineer the deal to determine the cash flows for a senior CMO. The cash flows for each month on an interest rate path are calculated using the scheduled principal, net interest, and prepayments for the collateral (i.e., the pool of agency pass-

throughs). The tranche's cash flows for each path are determined by the total principal and interest paid to the tranche, the interaction of the cash flow rules, and the prepayment model.

Step 3: Calculate the present value of cash flows for each interest rate path.

The present values of cash flows for each interest rate path are calculated by discounting the cash flows for each path by a discount rate. The discount rate is estimated using the simulated spot rates for each month on the interest rate path plus an appropriate spread. The simulated spot rates are determined from the simulated future monthly rates. The following equation quantifies the relationship that holds between the simulated spot rate, $z_T(n)$, for month T on path n, and the simulated future monthly rates, $f_j(n)$:

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)] \ldots [1 + f_T(n)]\}^{1/T} - 1$$

where:
$z_T(n)$ = simulated spot rate for month T on path n
$f_j(n)$ = simulated future one-month rate for month j on path n

The interest rate paths for the simulated future one-month rates are converted to the interest rate paths for the simulated monthly spot rates. The present value of the cash flows for month T on interest rate path n discounted at the simulated spot rate for month T, $z_T(n)$, plus a spread, K, is:

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where:
$PV[C_T(n)]$ = present value of cash flows for month T on path n
$C_T(n)$ = cash flow for month T on path n
$z_T(n)$ = spot rate for month T on path n
K = spread

The present value for path n is determined as the sum of the present values of the cash flows for each month on path n as follows:

$$PV[path(n)] = PV[C_1(n)] + PV[C_2(n)] + \ldots + PV[C_T(n)]$$

where:
$PV[path(n)]$ = present value of interest rate path n

Step 4: Calculate the theoretical value of the mortgage security.

The theoretical value for a specific interest rate path is thought of as the present value of all cash flows in that path, assuming that path was actually realized. The theoretical value of the mortgage security is calculated as the average present value of all theoretical values for each interest rate path as follows:

$$\text{theoretical value} = \frac{PV[path(1)] + PV[path(2)] + \ldots + PV[path(N)]}{N}$$

where:
N = number of interest rate paths

This average theoretical value is typically the only measurement that is evaluated when Monte Carlo simulations are used to value MBSs. It is unfortunate that other potentially valuable information, such as the distribution of the path present values, is usually ignored.

Option-Adjusted Spread

LO 47.9: Define Option Adjusted Spread (OAS), and explain its challenges and its uses.

The **option-adjusted spread** (OAS) is defined as the spread, *K*, that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The OAS is mathematically determined by the following relationship:

$$\text{market price} = \frac{\text{PV[path(1)]} + \text{PV[path(2)]} + \ldots + \text{PV[path(N)]}}{\text{N}}$$

where:
N = number of interest rate paths

The left-hand side of the equation is the current market price of the MBS. The right-hand side of the equation is the Monte Carlo model's output of the average theoretical value of the MBS. The OAS is determined with an iterative process. If the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

The OAS can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. In other words, the OAS is *option adjusted* because the cash flows on the interest rate paths take into account the borrowers' option to prepay. An investor could estimate the value of a security using the OAS for comparable bonds to determine whether or not to invest in the security. A second approach is to compare the OAS generated at the market price to those available for comparable securities or an investment benchmark (such as a cost of funds).

Cash flows for MBSs are monthly annuity payments, while Treasury securities pay semiannual interest-only payments and a large bullet payment. The **zero-volatility spread** (*z*-spread) is a spread measure that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. It is a more accurate measure because it compares an MBS to a portfolio of Treasury securities. The zero-volatility spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. Thus, an iterative process is required to determine the zero-volatility spread.

The zero-volatility spread accounts for variations in MBS principal payments at a given prepayment rate or speed. However, it does not consider the impact that prepayment risk or changing prepayment rates have on the value of the MBS.

The option cost measures the prepayment (or option) risk. It is the implied cost of the option embedded in the MBS. The option cost is calculated as the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread as follows:

option cost = zero-volatility spread – OAS

Therefore, the option cost is a by-product of the Monte Carlo analysis and is not determined using traditional option value approaches. As volatility declines, the option cost decreases, and the previously described relationship suggests that OAS increases as volatility declines, all other things equal.

OAS Challenges

There are four important limitations to consider when using OAS:

- Modeling risk associated with Monte Carlo simulations.
- Required adjustments to interest rate paths.
- An underlying assumption of a constant OAS over time in the model.
- The dependency of the underlying prepayment model.

The OAS is generated through Monte Carlo simulations. Therefore, the OAS is subject to all modeling risks associated with the simulation. Interest rate paths must be adjusted to ensure securities or rates making up the benchmark curve are properly valued when using Monte Carlo methods. This process of adjusting interest rate paths is subject to modeling error. If there is a term structure to the OAS, then this is not reflected in the Monte Carlo process because the OAS methodology assumes a constant OAS.

The prepayment model is very complex, given the amount of uncertainty regarding important variables. The behavior of both borrowers and lenders changes over time. Thus, the greatest weakness of using OAS valuation estimates generated from the Monte Carlo simulation is the dependence on the prepayment model.

Additionally, both *z*-spreads and OAS measures assume the securities are held to maturity. Some investors may hold a security to maturity, but many investors will only hold a security over a finite horizon. Thus, the investor should analyze the securities in a manner that is consistent with the investor's asset management horizon.

Key Concepts

LO 47.1

Key attributes that define mortgages are lien status, original loan term, credit classification, interest rate type, prepayments/prepayment penalties, and credit guarantees.

Agency MBSs are those that are guaranteed by government-sponsored enterprises (GSEs). Most of the MBSs are issued by these GSEs.

The GSEs have restrictions on which mortgages they can guarantee/securitize, which opened up the private label market (non-agency MBSs) for those participants willing to take on the risks inherent in nonconventional loans—jumbo loans and/or loans with high loan-to-value ratios.

LO 47.2

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. A level-payment, fixed-rate conventional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Even though the term, rate, and payment are fixed, the cash flows are not known with certainty because the borrower has the right to repay all or any part of the mortgage balance at any time.

LO 47.3

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

Other factors that influence prepayments include seasonality, age of mortgage pool, personal, housing prices, and refinancing burnout.

LO 47.4

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities (MBSs) to investors; the securities are backed by the mortgage loans as collateral.

Fixed-rate pass-through securities trade in one of the following ways:

- The specified pools market.
- The To Be Announced (TBA) market.

LO 47.5

The value of an MBS is a function of:

- Weighted average maturity (WAM).
- Weighted average coupon (WAC).
- Speed of prepayments.

Regarding prepayment speeds, the single monthly mortality (SMM) rate is derived from the conditional prepayment rate and is used to estimate monthly prepayments for a mortgage pool:

$$SMM = 1 - (1 - CPR)^{1/12}$$

LO 47.6

A dollar roll transaction occurs when an MBS market maker is buying positions for one settlement month and, at the same time, selling those same positions for another month.

LO 47.7

Borrowers may prepay a mortgage due to the sale of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur when the borrower has defaulted on the mortgage or when the borrower has cash available to make partial prepayments (curtailment).

LO 47.8

The Monte Carlo methodology is a simulation approach for valuing MBSs. The binomial model is not appropriate for valuing MBSs because MBSs have embedded prepayment options and the historical evolution of interest rates over time impacts prepayments.

A mortgage security is valued using the Monte Carlo methodology by simulating the interest rate path and refinancing path, projecting cash flows for each interest rate path, calculating the present value of cash flows for each interest rate path, and calculating the theoretical value of the mortgage security.

LO 47.9

The option-adjusted spread (OAS) is the spread that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The zero-volatility spread (*z*-spread) is the spread that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. The option cost is the implied cost of the embedded prepayment option and is calculated as the difference between the *z*-spread and OAS.

Four major limitations of OASs are related to: (1) modeling risk associated with Monte Carlo simulations, (2) required adjustments to interest rate paths, (3) model assumption of a constant OAS over time, and (4) dependency on the underlying prepayment model.

Concept Checkers

1. Which of the following factors is least likely to influence the level of residential mortgage prepayments?
   A. Seasonality.
   B. Inflation.
   C. Housing prices.
   D. Age of mortgage pool.

2. If the conditional prepayment rate (CPR) for a pool of mortgages is assumed to be 5% on an annual basis and the weighted average maturity of the underlying mortgages is 15 years, which of the following amounts is closest to the constant maturity mortality?
   A. 0.333%.
   B. 0.405%.
   C. 0.427%.
   D. 0.5%.

3. Which of the following factors would not cause a dollar roll to trade special?
   A. Decrease in the back month price.
   B. Increase in the front month price.
   C. Surplus of securities in the market used for settlement.
   D. Shortage of securities in the market used for settlement.

4. When using the Monte Carlo approach to estimate the value of mortgage-backed securities (MBSs), the model should:
   A. use one consistent volatility measure for all interest rate paths.
   B. use a short/long yield volatility approach.
   C. use annual interest rates over the entire life of the mortgage security.
   D. ignore the distribution of the interest rate paths used to determine the theoretical value.

5. All of the following describe limitations of using option-adjusted spreads (OASs) for valuing mortgage-backed securities (MBSs) except:
   A. modeling risk is associated with Monte Carlo simulations.
   B. model requires making adjustments to interest rate paths.
   C. model assumes a dynamic OAS over time.
   D. prepayment model influences the model valuation.

Concept Checker Answers

1. **B** Seasonality does impact the level of prepayments—they are noticeably higher in the summertime. Increases in housing prices may spur an increase in prepayments caused by refinancing mortgages stemming from borrowers wanting to take out some of the increased equity for personal use. The lower the age of the mortgage pool, the less likely the risk of prepayment.

2. **C** The constant maturity mortality (or single monthly mortality rate) is a monthly measure. Its relationship to CPR is as follows:

   $SMM = 1 - (1 - CPR)^{1/12} = 1 - (1 - 0.05)^{1/12} = 1 - 0.95^{1/12} = 0.43\%$

3. **C** When the drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading special. It could be caused by:
   - A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).
   - An increase in the front month price (due to an increased demand in the front month for deal collateral).
   - Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, which would increase the front month price.

4. **B** When using the Monte Carlo approach to estimate the value of MBSs, the model should use more than one volatility measure for all interest rate paths. It is very common to use a short/long yield volatility approach to estimate monthly rates. Although the information regarding the distributions of interest rate paths is oftentimes ignored, it contains valuable information and should be considered.

5. **C** When using OAS to value MBS, the model assumes a constant OAS over time. This is problematic if there is a term structure to the OAS because this is not reflected in the Monte Carlo process.

The following is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by GARP®. This topic is also covered in:

The Rating Agencies

Topic 48

Exam Focus

This topic discusses the relationship among ratings, investment market participants, and the regulatory process. The key points are: (1) uses of credit ratings by financial market participants, (2) the ratings performance for rated securities, and (3) examples of ratings-based regulations and relations between rating agencies and regulatory bodies. For the exam, be familiar with the ratings scales utilized by both Moody's and S&P. Also, understand the relationship between regulators and rating agencies.

Uses of Credit Ratings

LO 48.1: Describe the role of rating agencies in the financial markets.

The role of **credit rating agencies** is to evaluate the creditworthiness of debt securities issued by corporate and other obligors and also to evaluate the creditworthiness of the issuers. An agency's job is to inform investors of the likelihood that an issuer will pay the promised interest and principal payments from a security.

LO 48.2: Explain market and regulatory forces that have played a role in the growth of the rating agencies.

In the past, banks have been the primary source of debt capital. Today, the capital markets have largely replaced banks—public financial markets are the purchasers of debt securities. Public financial market participants have a wide range of expertise in evaluating credit, but they often lack the credit expertise of banks. In these markets, borrowers, investors, and regulators depend on ratings supplied by rating agencies in the following ways:

- Borrowers need credit ratings to assure access to capital and a reasonable cost of borrowing.
- Investors use credit ratings to estimate potential losses associated with their debt investments and to evaluate potential risk and return.
- Regulatory agencies use credit ratings to establish capital requirements for broker-dealers, banking and thrift institutions, and insurance companies. Margin requirements can also depend on credit ratings.

The first bond rating agency began about a century ago in the United States, and prospered well into the 1930s. (Credit rating agencies began even earlier, in the mid-19th century). Over time, the number of issuer ratings declined until the 1950s as the creditworthiness of issues improved. Since the 1950s, the role of rating agencies has increased dramatically for several reasons. The use of ratings for regulatory purposes became established and widespread. The number of corporate issuers increased. The breadth of instruments and

obligors spread far beyond bonds, to include asset-backed securities, commercial paper, municipal bonds, counterparty risk, insurance companies, and other credit risks.

Professor's Note: Nationally recognized statistical rating organizations (i.e., Moody's, Standard and Poor's, Fitch Ratings, etc.) are often referred to as "NRSROs."

Interpreting Credit Ratings

LO 48.3: Describe a rating scale, define credit outlooks, and explain the difference between solicited and unsolicited ratings.

LO 48.4: Describe Standard and Poor's and Moody's rating scales and distinguish between investment and noninvestment grade ratings.

A rating scale is a series of categories that can be applied to investments (or companies), ranking them from very creditworthy to in default. The investments in one category, typically, are less creditworthy than the next higher category and, similarly, more creditworthy than the next lower category. The rating scales for Moody's and Standard and Poor's are shown in Figure 1 to follow.

A credit outlook often accompanies a rating, indicating the likely direction of a future credit change. If the outlook is positive, neutral, or negative, the agency is signaling that the next change in rating would be to raise, leave unchanged, or lower the rating. If the outlook is developing/evolving, the rating agency is signaling that a change is likely, but it is unsure of the direction.

While investors (i.e., subscribers) used to pay the rating agencies for their services, issuers now pay for their ratings. Publicly registered securities are nearly always rated. In some cases, a rating agency will assign ratings without a request from the issuer. These ratings are called **unsolicited** (a.k.a. agency-initiated) **ratings**. If a rating is unsolicited, the rating agency will disclose this fact. Whether solicited or unsolicited, the rating agency will offer to meet with the issuer and give them the opportunity to appeal their ratings.

Moody's and Standard and Poor's credit ratings are the most widely known in the financial markets. In general, the ratings are divided into two main segments: investment grade and speculative grade. **Investment grade** security ratings indicate those firms with adequate repayment capacity. These firms are of the highest quality and have strong financial operations. Many institutional investors are restricted to making investments only in securities ranked as investment grade. The debt issues of firms rated in the **speculative grade** category are those instruments issued by firms with higher risks associated with their ability to repay obligations. Many investors are not allowed to purchase securities rated within the speculative grade category. Securities rated as investment grade sometimes receive ratings downgrades into the speculative rating category, which often necessitate a sale of those "**fallen angels.**"

Professor's Note: Recall the discussion of "fallen angels" from Topic 46.

Moody's and Standard and Poor's use the following systems to rate the credit risk of issuers. The general ratings given by each rating agency are listed from least credit risk to greatest credit risk.

Figure 1: Rating Scales

Investment Grade	*S&P*	*Moody's*
Highest quality	AAA	Aaa
High quality	AA	Aa
Strong capacity for repayment	A	A
Adequate capacity for repayment	BBB	Baa

Speculative Grade	*S&P*	*Moody's*
Likely to meet obligations with uncertainty	BB	Ba
High-risk obligations	B	B
Currently vulnerable to default	CCC	Caa
	CC	Ca
Lowest quality	C	C
In default	D	

Moody's appends numerical modifiers 1, 2, or 3 to each generic rating classification from Aa through Caa. Standard and Poor's has a similar practice of adding a "+" or a "–" to modify each rating from AA through CCC. Any issue rated BB or below from Standard and Poor's or Ba or below from Moody's is considered speculative grade. So, the demarcation between investment grade and speculative grade is between Baa and Ba for Moody's and BBB and BB for Standard and Poor's.

The Ratings Process

LO 48.5: Describe the difference between an issuer-pay and a subscriber-pay model and describe concerns regarding the issuer-pay model.

Rating agencies typically receive payment from issuers for their rating services. As mentioned earlier, in the **subscriber-pay model**, investors subscribed to the ratings agencies and paid for their services. The **issuer-pay** ("pay-for-rating") **model** is sometimes questioned as having the potential to distort the independence of the rating process. Evidence, however, indicates that this is not the case.

There is a symbiotic relationship between the ratings agencies and the firms receiving ratings that hinges on maintaining the highest independence and objectivity. The rating agency essentially acts as an external monitor of company activity and the accuracy of the ratings reflects the analytical capability of the rating company to measure credit risk. By selling its services, the rating agency is selling its reputation to analyze a firm's ability to repay its obligations. The firms receiving ratings also desire the highest reputation associated with the ratings agencies, because uncertainty in the ratings process would increase their cost of debt. Both parties want the utmost independence and highest reputation associated with the rating process, which serve to minimize any potential influence over the payment

mechanism while simultaneously maximizing rating process credibility. Regulators remain willing to use the ratings from NRSROs for a variety of regulatory purposes. The rating firms have a disciplined research process that is akin to disciplined academic research.

Corporate and Sovereign Debt Ratings

LO 48.6: Describe and contrast the process for rating corporate and sovereign debt and describe how the distributions of these ratings may differ.

The rating process will differ according to the type of instrument being rated. Additionally, there is variation in the process across rating agencies. The rating process for corporate bonds (following the example of S&P) focuses on the following areas[1]:

- Business risk.
- Industry characteristics.
- Competitive positioning.
- Management.
- Financial risk.
- Financial characteristics.
- Financial policies.
- Profitability.
- Capitalization.
- Cash flow protection.
- Financial flexibility.

Although the ratings agencies do not release their specific rating algorithms, both Moody's and Standard and Poor's indicate that they use industry and firm-specific inputs like those above when determining a firm's bond rating. Some of the factors incorporated in the industry component relate to industry characteristics and competitive factors. The firm specific factors relate to the financial condition of the company and are evaluated using ratio analysis. The weights for the various factors vary, with the heaviest weight on industry risk analysis. The firm's ratios are tracked through time and monitored for potential changes in firm default probability.

Internationally, the sovereign rating will be the ceiling for the rating of an issuer within that country. For sovereigns, there are additional factors to consider such as:

- Political stability.
- Social and economic coherence.
- Integration into global economic system.

Within a rating agency, the rating proposed for an issue/issuer is contrasted with other ratings in the same industry and firms in other industries. This is to assure that a rating (such as Baa) has the same meaning across firms and industries.

1 Caouette, John B., Edward I. Altman, and Paul Narayanan. 2008. *Managing Credit Risk*. John Wiley & Sons, Inc.

Corporate Bond Rating Performance

Ratings communicate an opinion about the creditworthiness of an issuer or an issuer's obligation. They should indicate the likelihood and severity of default. Characteristics of ratings performance for corporate bonds are as follows:

- Ratings and corporate default rates are inversely related. This inverse relationship holds for all time periods following the ratings, such as one year, five years, ten years, etc.
- Yield spreads over treasury bonds correlate highly with ratings (i.e., the greater the spread, the lower the credit rating).
- Default rates and ratings remain inversely related throughout the business cycle.
- Default rates for investment grade issues (Baa or better) are substantially lower than default rates for speculative grade issues (Ba or worse).
- Ratings do change, but are usually fairly stable. It is not uncommon for investment grade ratings to have the same rating at the end of the year that they had at the beginning of the year.

Ratings and Regulation

LO 48.7: Describe the relationship between the rating agencies and regulators and identify key regulations that impact the rating agencies and the use of ratings in the market.

Regulators like the high quality, independence, and widespread use of rating agency opinions. For rating agencies, the acceptance of their opinions by regulators is a strong signal of the quality of their work. Regulators accept the ratings of only a few rating agencies (NRSROs in the United States and the external credit assessment institutions [ECAIs] under Basel II).

In the United States, the Credit Rating Agency Reform Act of 2006 established the process for designating an NRSRO and provided SEC oversight (to assure the continuation of credible ratings). The Committee of European Banking Supervisors (CEBS) provides ECAI recognition, although it does not regulate or license the rating agencies. However, CEBS does apply criteria to the ratings such as objectivity, independence, international access and transparency, disclosure, resources, and credibility.

The relationship between regulators and rating agencies increases the benefits of ratings, which assist markets with reliable, convenient, and low-cost information. This relationship may make it difficult for new rating agencies to compete against the large, established agencies. There are some tensions between regulators and rating agencies about the interpretations of ratings. For example, the default rates of corporate bonds change over the business cycle, while some regulators would like the probabilities of default for a given rating category to be constant over this cycle. For rating agencies to do this, they would have to issue short-term rather than long-term ratings, and they would have to change ratings much more frequently.

LO 48.8: Describe some of the trends and issues emerging from the recent credit crisis relevant to the rating agencies and the use of ratings in the market.

Regulators and lawmakers are becoming more involved with rating agency activities. Economic problems have become considerably worse as of late. The recent economic turmoil should heighten the scrutiny of rating agencies. In addition, recent events have provided a rich environment for the rating agencies to reexamine and recalibrate their own methods.

Rating agencies are going to remain a major player in capital markets. The current level of uncertainty increases their relevance. Both economic and regulatory forces should accelerate changes in rating agencies—their roles, importance, and sophistication will probably all be enhanced.

Key Concepts

LO 48.1

Since many financial market participants may not be specialized credit analysts, they rely on credit ratings when making investments. Investors and regulators rely on credit ratings to judge not only risk-return relationships, but also appropriateness and suitability of potential investment instruments.

LO 48.2

Borrowers, investors, and regulators depend on ratings in the following ways:

- Borrowers need credit ratings to assure a reasonable cost of borrowing.
- Investors use credit ratings to estimate potential losses associated with their debt investments.
- Regulatory agencies use credit ratings to establish capital requirements for broker-dealers, banking and thrift institutions, and insurance companies.

LO 48.3

A rating scale is a series of categories that can be applied to companies, ranking them from very creditworthy to in default. A credit outlook often accompanies a rating, indicating the likely direction of a future credit change. In some cases, a rating agency will assign ratings without a request from the issuer (i.e., unsolicited ratings).

LO 48.4

Various ratings scales exist in the marketplace and carry a spectrum of ratings from high to low quality securities. Two popular rating scales have been created by Moody's and Standard and Poor's. In general, all rating scales are broken into two subcategories, investment grade and speculative grade, which indicate ability to repay obligations.

LO 48.5

Since rating agencies act as external monitors of operating companies, it is in the best interest of all involved for the raters to maintain independence and objectivity when assigning ratings. Although those being rated usually pay a fee for the rating service, the relationship among rating agencies, firms, and investors creates and requires the highest level of independence, which generates and enhances agency reputation.

LO 48.6

Ratings are generated by analyzing industry and firm-specific characteristics. Competitive conditions, as well as financial conditions of the company, are evaluated through time and compared across firms. Evidence indicates ratings are correlated to default rates and yield spreads, which are very critical to investors.

LO 48.7

Regulatory entities use ratings to determine eligible investments for regulated institutions. Regulators also use ratings to set capital requirements for financial institutions and to set margin requirements for various derivative securities.

LO 48.8

Regulators and lawmakers are becoming more involved with rating agency activities. Recent economic turmoil should heighten the scrutiny of rating agencies.

Concept Checkers

1. The demarcation between investment and speculative grade investments made by Moody's can be found between which of the following ratings?
   A. Aaa and Baa.
   B. A and Baa.
   C. Baa and Ba.
   D. Ba and Caa.

2. The independence, objectivity, and reputation of the rating agency are all enhanced by the:
   A. SEC Regulatory Guidelines.
   B. NRSRO status.
   C. synergistic relationship between financial market participants.
   D. ongoing payments made by issuers to ratings agencies.

3. A Moody's rating of A3 is roughly equivalent to a Standard and Poor's rating of:
   A. AA.
   B. A+.
   C. A–.
   D. AAA.

4. Which of the following statements is not consistent with the empirical evidence connecting ratings with issuance characteristics?
   A. Negative correlation between rating and price, all else equal.
   B. Negative correlation between rating and yield, all else equal.
   C. Negative correlation between rating and incidence of default, all else equal.
   D. Clear separation between investment grade and speculative security categories, all else equal.

5. Which of the following statements are not examples of how credit ratings are used by regulators?

   I. The Department of Labor may use credit ratings to determine which securities are eligible for investment by pension funds.
   II. Transactions involving some highly rated securities are exempt from certain securities reporting requirements.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Concept Checker Answers

1. **C** The separation between investment grade and speculative grade securities occurs between Moody's Baa and Ba ratings.

2. **C** All financial market participants, issuers, rating agencies, investors, and regulators benefit from having the highest level of independence and objectivity in the rating process.

3. **C** Both Moody's and Standard and Poor's use modifiers to clarify their generic ratings. A Moody's rating of A3 is equivalent to a Standard and Poor's rating of A–.

4. **A** There is a direct correlation between rating and price: the higher the rating, the higher the price.

5. **D** Both of these items are examples of how regulators use credit ratings. Note that this question is not addressed directly in the notes, but is an example of a "real world application" question.

Self-Test: Financial Markets and Products

15 Questions: 36 Minutes

1. An investor enters a short position in a gold futures contract at \$318.60. Each futures contract controls 100 troy ounces. The initial margin is \$5,000 and the maintenance margin is \$4,000. At the end of the first day the futures price rises to \$329.22. Which of the following is the amount of the variation margin at the end of the first day?
   A. \$0.
   B. \$62.
   C. \$1,000.
   D. \$1,062.

2. A large-cap U.S. equity portfolio manager is concerned about near-term market conditions and wishes to reduce the systematic risk of her portfolio from 1.2 to 0.90. Her portfolio value is \$56 million, and the S&P 500 futures index is currently trading at 1,050 and has a multiplier of 250. How can the portfolio manager's objective be achieved?
   A. Sell 47 contracts.
   B. Buy 47 contracts.
   C. Sell 64 contracts.
   D. Buy 64 contracts.

3. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-Note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-Note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments and discrete compounding. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

	2-year spot rate	3-year spot rate
A.	5.51%	5.92%
B.	5.46%	5.92%
C.	5.51%	6.05%
D.	5.46%	6.05%

4. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?
   A. If the pure expectations hypothesis of the term structure is correct, this decision will reduce the government's borrowing cost.
   B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.
   D. If the pure expectations hypothesis of the term structure is correct, this decision will increase the government's borrowing cost.

5. A portfolio manager owns Macrogrow, Inc., which is currently trading at $35 per share. She plans to sell the stock in 120 days but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a $0.50 per share dividend in 35 days and $0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is $27.50 and the risk-free rate has not changed, is closest to:
   A. $7.17.
   B. $7.50.
   C. $7.92.
   D. $7.00.

6. A 6-month futures contract on an equity index is currently priced at 1,276. The underlying index stocks are valued at 1,250 and pay dividends at a continuously compounded rate of 1.70%. The current continuously compounded risk-free rate is 5%. The potential arbitrage is closest to:
   A. 5.20.
   B. 8.32.
   C. 16.58.
   D. 26.00.

7. Company J and Company K enter into a 2-year plain vanilla interest rate swap. Company J agrees to pay Company K a periodic fixed rate on a notional principal over the swap's tenor. In exchange, Company K agrees to pay Company J a periodic floating rate on the same notional principal. Assume currency is the same, so the net payment will be exchanged. The exchanges will be made semi-annually. The reference rate is the 6-month LIBOR. The fixed rate of the swap is 1.1%, and the notional principal is $100 million. 6-month LIBOR rates are as follows:

Beginning of Period	*LIBOR*
1	0.5%
2	0.75%
3	1.00%
4	1.25%
5	1.50%

What is the net payment for the end of the first period?
A. Company J pays Company K $300,000.
B. Company J pays Company K $550,000.
C. Company K pays Company J $250,000.
D. Company K pays Company J $50,000.

Use the following information to answer Questions 8 and 9.

Stock ABC trades for $60 and has 1-year call and put options written on it with an exercise price of $60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is $4.09.

Chefron, Inc. common stock trades for $60 and has a 1-year call option written on it with an exercise price of $60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

8. The value of the put on ABC stock is closest to:
A. $1.16.
B. $3.28.
C. $4.09.
D. $1.00.

9. The value of the call on Chefron stock is closest to:
   A. \$3.51.
   B. \$4.16.
   C. \$5.61.
   D. \$6.53.

10. One of your clients, Christopher Stachowski, realizes that the market prices of options must take into account the beliefs of the market participants. He thinks he will be able to make significant profits because he believes that there will be a large movement in the direction of stock prices but is unsure which direction. Such a belief is completely different from the other market participants. As a result, Christopher would like you to implement an options trading strategy to generate him those profits. Which of the following combination option strategies is likely to benefit the least amount from a large positive or negative movement in the price of the underlying?
    A. Strip.
    B. Strap.
    C. Collar.
    D. Long strangle.

11. Consider a bearish option strategy of buying one \$50 put for \$7, selling two \$42 puts for \$4 each, and buying one \$37 put for \$2. All the options have the same maturity. Calculate the final profit per share of the strategy if the underlying is trading at \$33 at expiration.
    A. \$1 per share.
    B. \$2 per share.
    C. \$3 per share.
    D. \$4 per share.

12. You believe that a stock will increase in price and would like to buy a call option. You would like to choose the date during the option's term when the option payoff is determined. However, if the option payoff is greater at the option's maturity, you want to be paid this value. What type of option should you buy?
    A. Chooser option.
    B. Compound option.
    C. Shout option.
    D. Asian option.

13. Suppose the spot rate is 0.7102 USD/CHF. Swiss and U.S. interest rates are 7.6% and 5.2%, respectively. If the 1-year forward rate is 0.7200 USD/CHF, an investor could:
    A. not earn arbitrage profits.
    B. earn arbitrage profits by investing in USD.
    C. earn arbitrage profits by investing in CHF.
    D. only earn arbitrage profits by investing in a third currency.

14. Consider a U.K.-based company that exports goods to the EU. The U.K. company expects to receive payment on a shipment of goods in 60 days. Because the payment will be in euros, the U.K. company wants to hedge against a decline in the value of the euro against the pound over the next 60 days. The U.K. risk-free rate is 3% and the EU risk-free rate is 4%. No change is expected in these rates over the next 60 days. The current spot rate is 0.9230 £ per €. To hedge the currency risk, the U.K. company should take a short position in a Euro contract at a forward price of:
A. 0.9205.
B. 0.9215.
C. 0.9244.
D. 0.9141.

15. A level-payment, fixed-rate mortgage has the following characteristics:
- Term 30 years.
- Mortgage rate 9.0%.
- Servicing fee 0.5%.
- Original mortgage loan balance $150,000.

The monthly mortgage payment is:
A. $416.67.
B. $1,125.00.
C. $1,206.93.
D. $1,216.70.

Self-Test Answers: Financial Markets and Products

1. D The short position loses when the price rises.

   ($329.22 – $318.60) × 100 = 1,062 loss

   Margin account will change as follows: $5,000 – $1,062 = $3,938

   Variation margin of $1,062 is required because the balance has fallen below the maintenance margin level. This variation margin payment is required in order to restore the account back to the initial level.

   (See Topic 34)

2. C The portfolio manager wants to reduce exposure to systematic risk so she will want to sell S&P index futures. This will reduce the current beta to her target beta of 0.90.

   number of contracts = (target beta – current beta) × (portfolio value / futures value)

   number of contracts = (0.9 – 1.2) × [$56 million / (1,050 × 250)]

   number of contracts = –64 (i.e., sell 64 contracts)

   (See Topic 35)

3. C Here are the cash flows associated with the three bonds:

	0	1	2	3
1-year	–$95.2381	+$100		
2-year	–$100.9232	+$6	+$106	
3-year	–$102.6730	+$7	+$7	+$107

   To find Z_2, the 2-year spot rate:

$$\$100.9232 = \frac{\$6}{1.05^1} + \frac{\$106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\%$$

   To find Z_3, the 3-year spot rate:

$$\$102.6730 = \frac{\$7}{1.05^1} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\%$$

   (See Topic 36)

4. **B** If the pure expectations hypothesis of the term structure is correct, altering the maturity of the government's borrowing will not affect the government's borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government's borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

   (See Topic 36)

5. **A** The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

   $$\text{PVD} = \frac{\$0.50}{1.04^{35/365}} = \$0.4981$$

   $$\text{FP} = (\$35 - \$0.4981) \times 1.04^{120/365} = \$34.95$$

   $$\text{V}_{45}\,(\text{short position}) = -\left(\$27.50 - \frac{\$34.95}{1.04^{75/365}}\right) = \$7.17$$

   (See Topic 37)

6. **A** $F = S \times e^{(\text{risk-free rate} - \text{dividend yield}) \times t}$

   $F = 1{,}250 \times e^{(0.05 - 0.017) \times 0.5}$

   $F = 1{,}270.80$

   The actual futures price is 1,276, so selling the futures and buying the underlying index nets a profit of 1,276 – 1,270.80 = 5.20.

   (See Topic 37)

7. **A** Floating = \$100 million × 0.005 × 0.5 = \$250,000

   Fixed = \$100 million × 0.011 × 0.5 = \$550,000

   (See Topic 39)

8. **A** According to put/call parity, the put's value is:

   $$p_0 = c_0 - S_0 + \left(X \times e^{-R_c^f \times T}\right) = \$4.09 - \$60.00 + \left[\$60.00 \times e^{-(0.05 \times 1.0)}\right] = \$1.16$$

   (See Topic 41)

9. **A** ABC and Chefron stock are identical in all respects except Chefron pays a dividend. Therefore, the call option on Chefron stock must be worth less than the call on ABC (i.e., less than \$4.09). \$3.51 is the only possible answer.

   (See Topic 41)

10. **C** A collar is the combination of a protective put and a covered call. Ignoring transaction costs, at levels below the put strike price or above the call strike price, the profit from a collar levels off. Between the put strike price and the call strike price, the profit level is gradually rising.

    (See Topic 42)

11. **B** Consider each option separately:

\$50 long put:	\$50 – \$33 = +\$17
\$42 short put:	\$42 – \$33 = –\$9 × 2 = –\$18
\$37 long put:	\$37 – \$33 = +\$4
Net cost of options:	(–7 + 8 – 2) = –\$1
Overall profit per share:	\$2 per share

(See Topic 42)

12. **C** The shout option allows the buyer to choose the date when he "shouts" to the option seller that the intrinsic value should be determined. At expiration, the option buyer receives the maximum of the shout value or the intrinsic value at expiration.

(See Topic 43)

13. **C** Note that while the USD has the lower interest rate, it is also trading at a forward discount relative to the CHF. Since the USD will earn less interest *and* depreciate in value, we definitely want to invest in CHF (not in USD), and no calculation is necessary.

As an illustration of covered interest arbitrage, we have:

$$(1+R_A) < \frac{(1+R_B)(\text{forward rate})}{\text{spot rate}}$$

$$1.052 < \frac{(1.076)(0.72)}{0.7102} = 1.0908$$

Today:
(1) Borrow USD1 at 5.2% and purchase CHF at \$0.7102 to get \$1 / 0.7102 = 1.408 CHF at spot rate.
(2) Lend the purchased CHF at 7.6% and sell forward 1.5150 CHF at the forward rate of 0.7200 USD/CHF.

In one year:
(1) Use the proceeds of the savings account [(1.408)(1.076) = 1.5150 CHF] to purchase USD1.0908 at the forward rate (1.515 CHF × 0.72 USD/CHF).
(2) Pay off the loan of USD1 × 1.052 = USD1.052 and earn a riskless profit = USD1.0908 – USD1.052 = USD0.0388.

(See Topic 45)

14. **B** The U.K. company will be receiving euros in 60 days, so it should short the 60-day forward on the euro as a hedge. The no-arbitrage forward price is:

$$F_T = £0.923 \times \frac{1.03^{60/365}}{1.04^{60/365}} = 0.9215$$

(See Topic 45)

15. **C** N = 360; I = 9/12 = 0.75; PV = 150,000; CPT → PMT = $1,206.93

(See Topic 47)

Formulas

Financial Markets and Products

Topic 32

basis = $S_t - F_0$

where:
S_t = cash (or spot) price of the underlying asset at time t
F_0 = current price of the futures contract

Topic 33

call option payoff: $C_T = \max(0, S_T - X)$

put option payoff: $P_T = \max(0, X - S_T)$

forward contract payoff: payoff = $S_T - K$

where:
S_T = spot price at maturity
K = delivery price

Topic 35

hedge ratio: $HR = \rho_{S,F} \dfrac{\sigma_S}{\sigma_F}$

beta: $\dfrac{Cov_{S,F}}{{\sigma_F}^2} = \beta_{S,F}$

correlation: $\rho = \dfrac{Cov_{S,F}}{\sigma_S \sigma_F}$

hedging with stock index futures:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

adjusting the portfolio beta: $\text{number of contracts} = (\beta^* - \beta)\dfrac{P}{A}$

Topic 36

discrete compounding: $FV = A\left(1+\frac{R}{m}\right)^{m \times n}$

continuous compounding: $FV = Ae^{R \times n}$

forward rate agreement: cash flow (if receiving R_K) $= L \times (R_K - R) \times (T_2 - T_1)$
cash flow (if paying R_K) $= L \times (R - R_K) \times (T_2 - T_1)$

Topic 37

forward price: $F_0 = S_0 e^{rT}$

forward price with carrying costs: $F_0 = (S_0 - I)\, e^{rT}$

forward price when the underlying asset pays a dividend: $F_0 = S_0 e^{(r-q)T}$

Topic 38

accrued interest = coupon $\times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$

cash price of a bond: cash price = quoted price + accrued interest

annual rate on a T-Bill: T-bill discount rate = $\frac{360}{n}(100 - Y)$

cheapest-to-deliver bond: quoted bond price – (QFP × CF)

Eurodollar futures price = \$10,000[100 – (0.25)(100 – Z)]

convexity adjustment:

actual forward rate = forward rate implied by futures – $(0.5 \times \sigma^2 \times t_1 \times t_2)$

duration-based hedge ratio: $N = -\frac{P \times D_P}{F \times D_F}$

Topic 39

forward rate between T_1 and T_2: $R_{forward} = R_2 + (R_2 - R_1)\,\frac{T_1}{T_2 - T_1}$

Topic 41

put-call parity:

$S = c - p + Xe^{-rT}$
$p = c - S + Xe^{-rT}$
$c = S + p - Xe^{-rT}$
$Xe^{-rT} = S + p - c$

lower and upper bounds for options:

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Topic 42

bull call spread: $\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$

bear put spread: $\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$

butterfly spread with calls:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

straddle: $\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$

strangle: $\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

Topic 44

pricing a commodity forward with a lease payment: $F_{0,T} = S_0 e^{(r - \delta_l)T}$

commodity forward pricing with storage costs: $F_{0,T} = S_0 e^{(r+\lambda)T}$

commodity forward pricing with convenience yield: $F_{0,T} = S_0 e^{(r-c)T}$

Topic 45

interest rate parity: $\text{forward} = \text{spot}\left[\frac{(1 + r_{DC})}{(1 + r_{FC})}\right]^T$

$$\text{forward} = \text{spot} \times e^{(r_{DC} - r_{FC})T}$$

nominal interest rate:

exact methodology: $(1 + r) = (1 + \text{real } r)[1 + E(i)]$

linear approximation: $r \approx \text{real} + E(i)$

Topic 46

original-issue discount (OID) = face value – offering price

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Topic 47

single monthly mortality rate: $SMM = 1 - (1 - CPR)^{1/12}$

option cost = zero-volatility spread – OAS

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Notes